

# Escuela de Verano en Matemáticas Discretas

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## Charla 1

Monday 6

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### A fully polynomial time additive approximation scheme for the envy minimizing Santa Claus problem<sup>1</sup>

Moritz Buchem

Maastricht University  
m.buchem@maastrichtuniversity.nl

We consider the so-called *envy minimizing Santa Claus problem*. In the standard Santa Claus problem as defined, Santa Claus needs to divide  $n$  gifts (jobs) over  $m$  children (machines). Gift  $j$  has value  $p_{ij}$  to child  $i$  and when child  $i$  receives a set of gifts  $S_i$ , then its utility is  $\sum_{j \in S_i} p_{ij}$ . The goal is to maximize the minimum utility over all children. In the envy minimizing variant, the goal is to distribute the gifts as to minimize the difference between the maximum utility and minimum utility, i.e.  $\max_i \sum_{j \in S_i} p_{ij} - \min_i \sum_{j \in S_i} p_{ij}$ . It can easily be seen that determining whether or not there is no envy is NP-complete by a reduction from 3-Partition or Partition in the case when  $m$  is not part of the input. As there cannot exist any polynomial time approximation algorithm and, therefore also no (F)PTAS for these type of problems, we consider the concept of a (fully) polynomial time *additive* approximation scheme (F)PTAdAS. We show that the envy minimizing Santa Claus problem is solvable in pseudo-polynomial time with a dynamic program and derive a FPTAdAS from this dynamic program. Given an  $\epsilon > 0$ , this FPTAdAS finds a solution to the envy minimizing Santa Claus problem with an additive performance guarantee of  $\epsilon p_{max}$  in  $O(\frac{n^{2m+1}}{\epsilon^m})$ .

This is joint work with José Verschae and Tjark Vredeveld.

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<sup>1</sup>Talk will be in English.

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## Charla 2

Tuesday 7

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### Clustering in a hyperbolic model of complex networks<sup>2</sup>

Markus Schepers

University of Groningen  
m.f.schepers@rug.nl

In this talk, we consider the clustering coefficient and clustering function in a random graph model proposed by Krioukov et al. in 2010. In this model, vertices are chosen randomly inside a disk in the hyperbolic plane and two vertices are adjacent if they are at most a certain hyperbolic distance from each other. It has been previously shown that this model has various properties associated with complex networks, including a power-law degree distribution, “short distances” and a non-vanishing clustering coefficient. The model is specified using three parameters: the number of vertices  $n$ , which we think of as going to infinity, and  $\alpha, \nu > 0$ , which we think of as constant. Roughly speaking, the parameter  $\alpha$  controls the power-law exponent of the degree distribution and  $\nu$  the average degree.

Here we show that the clustering coefficient tends in probability to a constant  $\gamma$  that we give explicitly as a closed-form expression in terms of  $\alpha, \nu$  and certain special functions. This improves over earlier work by Gugelmann et al., who proved that the clustering coefficient remains bounded away from zero with high probability, but left open the issue of convergence to a limiting constant. Similarly, we are able to show that  $c(k)$ , the average clustering coefficient over all vertices of degree exactly  $k$ , tends in probability to a limit  $\gamma(k)$  given explicitly as a closed-form expression in terms of  $\alpha, \nu$  and certain special functions. We are able to extend this last result also to sequences  $(k_n)_n$  where  $k_n$  grows as a function of  $n$ . Our results show that  $\gamma(k)$  scales differently, as  $k$  grows, for different ranges of  $\alpha$ . More precisely,  $\gamma(k) = \Theta(k^{2-4\alpha})$  if  $\frac{1}{2} < \alpha < \frac{3}{4}$ ,  $\gamma(k) = \Theta(\log(k)/k)$  if  $\alpha = \frac{3}{4}$  and  $\gamma = \Theta(k^{-1})$  if  $\alpha > \frac{3}{4}$ . These results contradict a prediction of Krioukov et al., which stated that the limiting values  $\gamma(k)$  should always scale with  $k^{-1}$  irregardless of the value of  $\alpha$ .

Joint work with: Nikolaos Fountoulakis, Pim van der Hoorn, Tobias Müller

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<sup>2</sup>Talk will be in English.

# Escola de Verano en Matemáticas Discretas

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## Charla 3

Wednesday 8

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### Sample Complexity in Graph Problems<sup>3</sup>

Alane Marie De Lima

Federal U. of Paraná  
amlima@inf.ufpr.br

In this talk we present two randomized algorithms for problems in graph theory using tools of sample complexity. We first show an algorithm for estimating the percolation centrality in a directed weighted graph  $G$  in time  $\mathcal{O}(m \log^2 \text{diam}(G))$ , where  $\text{diam}(G)$  is the diameter of  $G$ . The estimate obtained by the algorithm is within  $\epsilon$  of the exact value with probability  $1 - \delta$ , for fixed constants  $0 < \epsilon, \delta \leq 1$ . The second algorithm presented in this talk is for the all-pairs shortest paths problem (APSP) for undirected graphs with non negative real edge weights. The algorithm presented here is for a variation of the APSP that fits neither in the exact nor in the approximate case. More precisely, for every pair of vertices of  $G$  the algorithm either computes the exact shortest path or does not compute any shortest path, depending on a certain measure of “importance” of a shortest path between the pair of vertices  $(u, v)$  in question, called shortest path centrality. The algorithm outputs a  $n^2$  matrix and runs in  $O(n^2 \log n)$ . For any fixed constants  $0 < \epsilon, \delta \leq 1$ , a shortest path between every pair of vertices  $u$  to  $v$  is computed with probability at least  $1 - \delta$  whenever the shortest path centrality of  $(u, v)$  is at least  $\epsilon$ .

This is joint work with my advisors Murilo V. G. da Silva and Andr L. Vignatti.

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<sup>3</sup>Talk will be in English.

# Escuela de Verano en Matemáticas Discretas

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## Charla 4

Thursday 9

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### Polytopality of the Cartesian Product of Graphs

Steffania Sierra Galvis

Universidad Nacional de Colombia  
ssierrag@unal.edu.co

In this talk, we present open problem in combinatorics of polytopes associated with the Cartesian product of non-polytopal graphs. We aim to give a bibliographic review of the most important results about the polytopality of the graph products as a contextualization of the open problem, cited by Ziegler (*DocCourse Combinatorics and Geometry, pages 9–49, 2009*, and which was presented by Vincent Pilaud at the Combinatorics meeting *Algebra, Geometry and Optimization – ECCO 2018*).