Optimización en economía forestal

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Introduction

- Find the economically optimal policy, using a dynamic model of a forest plantation

- Study its properties: sustainability, asymptotic convergence.

- In the literature of natural resources, we find:

  natural renewable resources whose growth depends mainly on remaining population.

- Forests are different:

  growth depends largely on available land.

- If trees can be planted:

  zero is not an steady state.

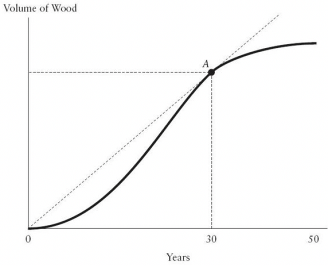
First model

Simplifying assumptions

- All trees are **identical and harvested at the same time**.

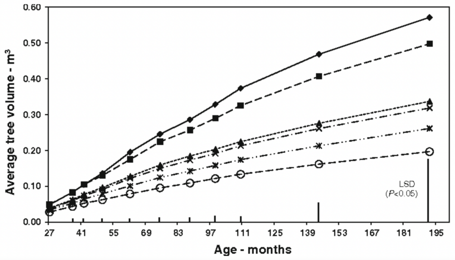
- The timber volume of the tree increases according to the *biomass function* $f(t)$.

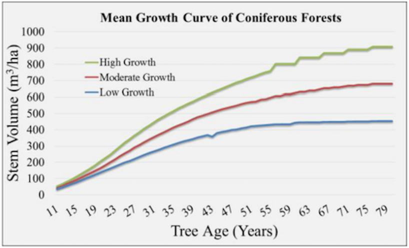
$\Rightarrow$ **timber content of the forest depends only on trees' age.**



- The price p per unit of timber is **known and constant** and not influenced by the age of the tree.

- We are **not** considering any environmental externalities such as: flood control, pollution abatement, species preservation, tourism, etc.


0.60
0.50
0.40
0.30
0.20
0.10
0.00
Average tree volume - m³
27
41
55
69
83
97
111
125
139
153
167
181
195
Age - months
LSD
(P<0.05)


Mean Growth Curve of Coniferous Forests
1000
900
800
700
600
500
400
300
200
100
0
Stem Volume (m³/ha)
High Growth
Moderate Growth
Low Growth
11
15
19
23
27
31
35
39
43
47
51
55
59
63
67
71
75
79
Tree Age (Years)

We plant at $t = 0$, when should we harvest?

$$\max_{a \in \mathbb{N}} \quad b^a \, p \, f(a) \tag{1}$$

$$\max_{a \in \mathbb{N}} \quad \frac{b^a}{1 - b^a} \, p \, f(a) \tag{2}$$

- Faustmann 1849, proposed the correct solution.
- Remained unknown to mainstream economists until 1940 and even later!

Outline

Mitra and Wan model (1985, 1986)

Classify tress according to age:

age classes

- Important assumptions:
  - • same age $\Rightarrow$ identical
  - • number trees $\propto$ surface

  (managed forest plantations)

$\Rightarrow$ **timber content$\big/$unit of forest area depends only on trees' age.**

Mitra and Wan model

State at year t

$$\begin{pmatrix} x_{N,t} \\ x_{N-1,t} \\ \vdots \\ \vdots \\ x_{2,t} \\ x_{1,t} \end{pmatrix}$$

$x_{a,t}$: area occupied by trees of age a at year t

S: total area ($S = \sum_{a=1}^{N} x_{a,t}$)

$N+1$: age the trees die.

Mitra and Wan model

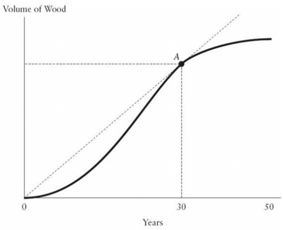
Each year we decide what to harvest of each age class.

Toy example: $N = 4, \;\; S = 10$

$$\begin{array}{cccccc} \text{State today} & \begin{pmatrix} 1 \\ 3 \\ 0 \\ 6 \end{pmatrix} & & & & \\ & & \Longrightarrow & \begin{pmatrix} 0 \\ 1 \\ 0 \\ 5 \end{pmatrix} & \Longrightarrow & \begin{pmatrix} 1 \\ 0 \\ 5 \\ 4 \end{pmatrix} \\ \text{Harvesting decision} & \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} & & \text{right after harvesting} & & \text{State tomorrow} \end{array}$$

Mitra and Wan model

Biomass coefficients



Biomass function

f_a : timber volume per unit of area.

We harvest: $(c_N, \ldots, c_2, c_1)^T$

$\Rightarrow$ total timber volume $c = \sum_a f_a c_a$

In the example: harvest $= (1, 2, 0, 1)^T$

$\Longrightarrow c = 1 \times f_4 + 2 \times f_3 + 1 \times f_1$

- c provides benefits equal to $u(c)$

Optimization problem

$$
(\mathrm{P}_{X_0}) \begin{cases} \max & \sum_{t\in\mathbb{N}} b^t u(c_t) \\ \text{s.a.} & c_t = \sum_{a=1}^{N} f_a c_t^a \\ & x_{t+1}^{a+1} = x_t^a - c_t^a \qquad a = 1, \ldots, N-1 \\ & c_t^N = x_t^N \\ & \sum_{a=1}^{N} x_t^a = S \\ & x_0^a = X_0^a \end{cases}
$$

Comments:

ℓ^∞, linear constrains, Weierstrass Theorem (weak* topology)

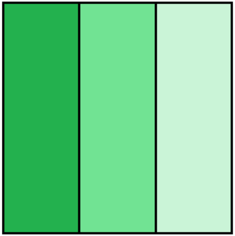
No dual solution can be assured in general.

Outline

Sustainable states

Example: $N = 4$, $\sigma = 3$.

$$\hat{\mathbb{X}} = \left(0, \frac{S}{3}, \frac{S}{3}, \frac{S}{3}\right).$$



- Constant harvest: $f_3 S/3$
- Constant structure of the trees plantation

Sustainable states

$$\hat{\mathbb{X}} = \Bigg(\underbrace{0, \ldots, 0}_{N-\sigma}, \underbrace{\frac{S}{\sigma}, \ldots \ldots, \frac{S}{\sigma}}_{\sigma}\Bigg)$$

"[...] there has been a tradition in forestry management which claims that the goal of good policy is to have sustained forest yield, or even maximum sustained yield somehow defined."

Samuelson, Economics of forestry in an evolving society, Econ. Inquiry XIV (1976).

Mitra and Wan (1985, 1986)

- There exists one sustainable state such that if the forest's initial state is the sustainable state, it is optimal to remain there (Foresters would be happy).

  This sustainable state is an steady state.

- If the initial forest is not the steady state:

  convergence to the sustainable state? $\boxed{\text{No!}}$

Results

Salo & Tahvonen (2002, 2004)

- Characterize steady state.
- Optimal periodic rotation: steady state is stable but **not** asymptotically stable!
- Conjecture: convergence to a periodic trajectory.
  - Solved for a 2-dimensional problem
  - Numerical simulation support the conjecture

Outline

Undiscounted future

- A usual objective function (discounting future utilities),

$$\sum_{t=1}^{\infty} b^t u(c_t), \qquad \text{with } b \in (0,1).$$

- We consider the undiscounted case $(b = 1)$
  - Technical interest
  - Continuity with respect to discount factor
  - Intergenerational equity issues

$b = 1$

$$\sum_{t=1}^{\infty} u(c_t) = ? \quad \text{Useless!!} \qquad \Rightarrow \qquad \lim_{T \to \infty} \sum_{t=1}^{T} [u(c_t) - u(c_t^*)] = ?$$

Undiscounted future

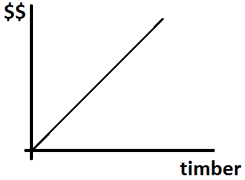
We compare every pair of possible trajectories from $\mathbb{X}(0)$

A trajectory $\{\mathbb{X}^*(t)\}$ is *optimal* if for every other trajectory from $\mathbb{X}(0)$:

$$\limsup_{T\to\infty} \sum_{t=0}^{T}[u(c(t)) - u(c^*(t))] \leq 0$$

Results (Mitra and Wan 1986)

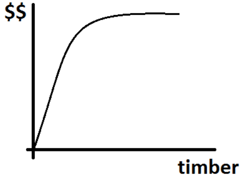
Benefit functions



Linear

∃! optimal trajectory:

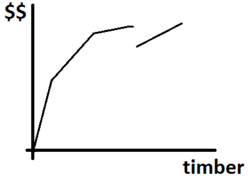
periodic



Strictly concave

∃! optimal trajectory:

converges
to the steady state

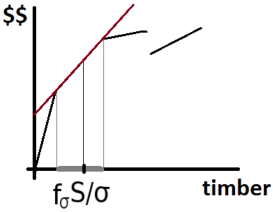


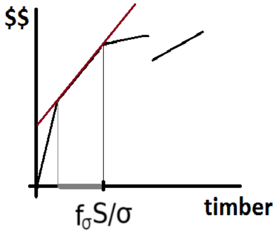
General

Optimal trajectory:

?

Non-interiority condition: Khan-P. (2012)





Cond. does not hold:

- Convergence to a periodic cycle
- Linear case

Cond. holds:

- Convergence to the steady state
- Strictly concave case

Open questions:

[-] Apply techniques to discounted future?

[-] Apply to other models?

Outline

1. Tree farm representation
2. Results and open problems
   - Steady state and stability
   - No discounting
   - Deforestation

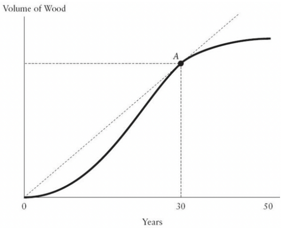
Introduction

- Generally, deforestation is associated to:
    - myopic management practices
    - weak property rights
- Our model of a managed plantation has
    - infinite time horizon
    - properties rights strongly enforced
- We will see that deforestation might be optimal.
- We introduce degrees of *irreversibility of deforestation*, i.e., cost of reforesting a depleted area.

The model (Salo & Tahvonen, 2004)

The state of the forest is:

$$\begin{pmatrix} x_N \\ \vdots \\ x_2 \\ x_1 \end{pmatrix} y$$



Biomass function

f_a : timber volume / area

Figure taken from doi:10.3390/rs71115082

- x_a: area occupied by trees of age a (age class a),
- y: area dedicated to alternative use,
- $\sum_a x_a + y \leq S$: total area of land.

The model: dynamics (toy) example

Each year we decide what to harvest of each age class,
and how to allocate all the liberated land.

Toy example: $N = 4$, $S = 10$

State today $\begin{pmatrix} 1 \\ 3 \\ 0 \\ 2 \end{pmatrix} 4$

Harvesting decision $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$

$\Longrightarrow$ $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} 4$ right after harvesting

$\Longrightarrow$ $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2.5 \end{pmatrix} 5.5$ State tomorrow

Optimization problem

- utility of volume of timber harvested: $u(c_t)$.
- utility of alternative use: $w(y_t)$.

- costs of plantation: $p\ x_{1,t+1}$
- reforestation costs: $q\ (X_{t+1} - X_t)$ if $(X_{t+1} - X_t) > 0$ and 0 otherwise.

$$(P) \left\{ \begin{array}{ll} \max & \sum_{t=0}^{\infty} b^t \left(u(c_t) + w(y_t) - p x_{1,t+1} - q[X_{t+1} - X_t]_+\right) \\ \text{s.t.} & \text{constrains describing dynamics} \\ & \text{initial state given.} \end{array} \right.$$

X_t: total area covered by trees

b: discount factor

Total deforestation

Theorem (P & Roy 2015)

The optimal policy is such that new trees are **never** planted,
(leading to total deforestation from every initial state), if and only if,

$$(\mathbf{D}) \qquad \frac{b}{1-b} w'(S) \geq \frac{b^a}{1-b^a} f_a u'(0) - \frac{p}{1-b^a} \quad \forall a$$

Encourage deforestation

- High marginal returns on alternative use
- High planting costs

Encourage conservation

- High price for timber
- High timber content of trees

Introducing reforestation costs (P& Roy 2019):

Reforestation costs do not affect optimality of deforestation!!

Irreversibility of deforestation

Theorem (P & Roy 2015)

The optimal policy is such that new trees are **never** planted,
(leading to total deforestation from every initial state), if and only if,

$$(\mathbf{D}) \qquad \frac{b}{1-b} w'(S) \geq \frac{b^a}{1-b^a} f_a u'(0) - \frac{p}{1-b^a} \quad \forall a$$

Theorem (P&Roy 2019)

Total forest cover is non-increasing if

$$(\mathbf{M}) \quad \frac{b}{1-b} w'(S) + q \geq \frac{b^a f_a}{1-b^a} u'(0) - \frac{p}{1-b^a} \quad \forall a$$

Corollary

If the reforestation cost (q) is large enough there is

irreversibility of deforestation!

Irreversibility of deforestation

Theorem (P & Roy 2015)

The optimal policy is such that new trees are **never** planted,
(leading to total deforestation from every initial state), if and only if,

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Theorem (P&Roy 2019)

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Corollary

If the reforestation cost (q) is large enough there is

irreversibility of deforestation!

It total deforestation is not optimal

Theorem (Weak conservation)

If $q = 0$ there exists $\tilde{X}_a, a = 1 \ldots, N$ (implicitly characterized) such that

$$X_t \geq \tilde{X}_a \text{ at least once every } a \text{ years}$$

And, deforestation is not optimal iff $\exists\ a$ (at least one) such that $\tilde{X}_a > 0$.

Does not rule out a persistent oscillation of the forest cover.

Theorem (Strong conservation)

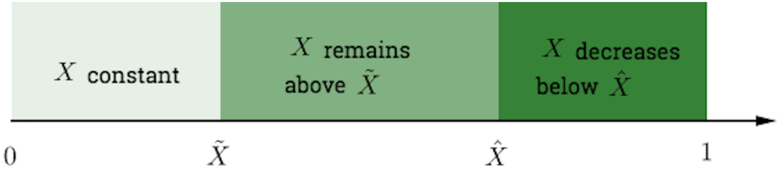
- *The bounds $(\tilde{X}_a)$ do not depend on reforestation costs (q).*
- *If $q > 0$ is large enough to have irreversibility and X_o is above $\tilde{X} = \max_a \tilde{X}_a$*

$$X_t \geq \tilde{X} \text{ for every } t$$

If deforestation is not optimal

Assume (**M**) holds (X_t is decreasing) and let $\tilde{X} = \max_{1,\dots,N}\{\tilde{X}_a\} > 0$

- If the initial forest cover is **above** $\tilde{X}$, then $X_t \geq \tilde{X} \quad \forall t$
- It is also possible to see that:
  - If the initial forest cover is **below** $\tilde{X}$, then $X_t = X_0 \quad \forall t$
  - There is $\hat{X} > \tilde{X}$ such that if the initial forest cover is **above** $\hat{X}$, then $\lim_t X_t \leq \hat{X}$



Conclusions

In this talk we:

- talked about the modelling specifics of forests as a natural renewable resource.
- presented some of the most important results and some open problems.

Conclusions: results

- There is a unique steady state. It is stable but not asymptotically stable.
- In the undiscounted framework, the mathematical tools are completely different.
  - There is a unique steady state.
  - It is locally stable.
  - It is asymptotically stable depending on the benefit function.
- Regarding deforestation,
  - we presented necessary and sufficient conditions under which total deforestation is optimal.
  - Showed that a high reforestation cost discourages expansion of forests but at the same assures a minimal forest cover.

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Muchas gracias!