

Abstracts  
Paralel Session Control and Inverse Problems in PDE  
FLACAM2019

**Parallel session W1: Control and Inverse Problems in PDE's Session 1**  
**Wednesday, Nov 6<sup>th</sup>, 11h00-12h30, Room B04**  
**Chair: Jaime Ortega**

Sebastián Zamorano  
Universidad de Santiago de Chile, Chile.

***Null controllability from the exterior of a one-dimensional nonlocal heat equation.***

**Abstract:** In this talk we discuss the null controllability problem for the one dimensional heat equation associated to the fractional Laplace operator of order  $s$  in the interval  $(0,1)$ . We will show that there exists a control function, which is localized in a nonempty open set at the exterior of the domain, such that the system is null controllable at any positive time if and only if  $s$  in the interval  $(1/2, 1)$ . This is a joint work with Mahamadi Warma from University of Puerto Rico.

Charles Dapogny  
Université de Grenoble-Alpes, France

**Abstract:** The purpose of this joint work with G. Allaire, F. Feppon and P. Frey is to introduce a robust front-tracking method for dealing with arbitrary motions of shapes, even dramatic ones (e.g. featuring topological changes); although this method is illustrated in the particular context of shape optimization, it naturally applies to a wide range of inverse problems and reconstruction algorithms. The method combines two different means of representing shapes: on the one hand, they are meshed explicitly, which allows for efficient mechanical calculations by means of any standard Finite Element solver; on the other hand, they are represented by means of the level set method, a format under which it is easy to track their evolution. The cornerstone of our method is a pair of efficient algorithms for switching from either of these representations to the other. Several numerical examples are discussed in two and three space dimensions, in the 'classical' physical setting of linear elastic structures, but also in more involved situations involving e.g. fluid-structure interactions.

Cristhian Montoya  
Universidad Técnica Federico Santa María, Chile.

***Simultaneous robust control and hierarchic control in some PDEs.***

**Abstract:** In recent works, the notion of searching for a robust control system is developed simultaneously with a strategy on hierarchic control. From a mathematical point of view, the robustness of a system is equivalent to find a saddle point because we are looking for maximizing the perturbation and simultaneously minimizing the control that stabilizes the system. In addition to the above, a hierarchic control strategy appears at the system. The scheme is based in considering a robust control problem for the follower control and its associated disturbance function. Afterwards, we consider the

notion of Stackelberg optimization (which is associated to the leader control) in order to deduce a controllability result for nonlinear systems.

**Parallel session W2: Control and Inverse Problems in PDE's Session 2**  
**Wednesday, Nov 6<sup>th</sup>, 14h00-15h30, Room B04**  
**Chair: Eduardo Cerpa**

Exequiel Mallea  
Universidad de Tarapacá, Chile

*A regularity criterion for a 3D chemo-repulsion system and its application to a bilinear optimal control problem.*

**Abstract:** A bilinear optimal control problem associated to a 3D chemo-repulsion model with linear production term is studied. The existence of weak solution is proved and a regularity criterion to get global in time strong solutions is established. As a consequence, the existence of a global optimal solution with bilinear control is deduced and, using a Lagrange multipliers theorem, first-order optimality conditions for local optimal solutions are derived.

Roberto Morales  
Universidad de Chile, Chile

*On the controllability of some PDE's with dynamic boundary conditions.*

**Abstract:** While controllability of PDE's with Dirichlet, Neumann and Fourier boundary conditions have been intensely studied in the past years, the same question for this kind of models with dynamic boundary conditions remains open. The aim of this talk is to present some recent results on the null controllability of parabolic and Schroödinger equations with boundary conditions of Wentzell type. Following the well-known duality between controllability and observability, these results can be obtained by a suitable observability inequality associated to the adjoint system of these models. In the same spirit of the classical ideas of A. Fursikov and O. Imanuvilov, the proof of these inequalities is based on suitable Carleman estimates for some PDE's with Wentzell boundary conditions by using appropriate weight functions.

Nicolás Carreño  
Universidad Técnica Federico Santa María, Chile

*Boundary null-controllability of a system coupling fourth- and second-order parabolic equations.*

**Abstract:** In this talk we will consider a control system coupling fourth- and second-order parabolic equations. We begin with a brief overview of the moment method and then we show some control properties of this system when we only control the second-order partial differential equation through a boundary condition. Actually, depending on the choice of the diffusion coefficients, we obtain positive and negative results for approximate- and null-controllability. In particular, we prove that for any given positive time  $T_0$ , we can find some diffusion coefficients such that the system is null-

controllable in time  $T$  if  $T > T_0$  and is not null-controllable if  $T < T_0$ . This is a joint work with Eduardo Cerpa and Alberto Mercado.

**Parallel session Th1: Control and Inverse Problems in PDE's Session 3**  
**Thursday, Nov 7<sup>th</sup>, 11h00-12h30, Room B04**  
**Chair: Carlos Conca**

François Murat  
Sorbonne Université and CNRS, France

***Homogenization of the Neumann's brush problem***

**Abstract:** In this lecture, I will describe joint work with Antonio Gaudiello (Naples, Italy) and Olivier Guibé (Rouen, France). We consider a sequence of domains  $\Omega^\varepsilon$  which have the form of brushes (in dimension  $N = 3$ ) or of combs (in dimension  $N = 2$ ). Each domain  $\Omega^\varepsilon$  is an open subset of  $\mathbb{R}^N$  with  $N \geq 2$  which is made of teeth distributed over a basis. When  $\varepsilon$  tends to zero, the basis remains fixed, while the teeth vary in size, forms and distributions. No periodicity is assumed on the distributions of the teeth, which are assumed to be vertical, cylindrical, and of fixed height, and do vary with  $\varepsilon$ . For each domain  $\Omega^\varepsilon$ , the diameter of every tooth is assumed to be less than or equal to  $\varepsilon$ ; the cross sections of the teeth, which are open, can vary from one tooth to another one; they are not assumed to be smooth, and the teeth can be adjacent, i.e. they can share parts of their boundaries. Finally the sequence of the characteristic functions of the cross sections of the teeth is assumed to have, as  $\varepsilon$  tends to zero, an  $L^\infty(\mathbb{R}^{N-1})$  weak-star limit  $\theta = \theta(x')$  (this latest assumption is an innocuous one). For this sequence of domains we study the asymptotic behavior, as  $\varepsilon$  tends to zero, of the solution of a linear second order elliptic equation with a zeroth order term which is bounded from below away from zero, and with a source term in  $L^2$ , when the homogeneous Neumann boundary condition is imposed on the boundaries of the domains  $\Omega^\varepsilon$ . This is a classical homogenization problem since the pioneering work presented by R. Brizzi and J.-P. Chalot in their Ph.D. Thesis in 1978, but our work takes place in a geometry which is much more general than the ones which have been considered since that time. In our paper A. Gaudiello, O. Guibé & F. Murat, Homogenization of the brush problem with a source term in  $L^1$ , published in Archive for Rational Mechanics and Analysis, volume 225, (2017), pp. 1-64, we have treated the case where  $\theta_0 \leq \theta(x') \leq 1$  for some  $\theta_0 > 0$  and given for the first time a corrector result. I will state and prove these homogenization and corrector results. If time permits, I will also describe a very recent work in which we treat the case where  $\theta_0 = 0$ , namely the case where one can have a cylindrical measurable subset of the zone of the teeth where there is no matter remaining at the limit.

Sergio Gutiérrez  
Pontificia Universidad Católica de Chile, Chile

***Optimal design under uncertainty using small amplitude homogenization***

**Abstract:** In many optimal design problems there is a certain degree of uncertainty, for example, in the source terms, the material properties, etc. We apply the small amplitude homogenization idea to solve the worst case scenario, which becomes explicit and then allows us to derive efficient numerical algorithms. The method, which is quite general, is presented together with numerical examples, including some standard cases like compliance minimization and the torsion problem.

Nicolas Lebbe  
Université de Grenoble-Alpes, France

***Robust shape optimization for nanophotonics***

**Parallel session Th2: Control and Inverse Problems in PDE's Session 4**  
**Thursday, Nov 7<sup>th</sup>, 12h30-15h30, Room B04**  
**Chair: Axel Osses**

Rodrigo Lecaros  
Universidad Técnica Federico Santa María, Chile

***An inverse problem for Moore–Gibson–Thompson equation arising in high intensity ultrasound***

Gino Montesinos  
Universidad de Aysen, Chile

***A simplified Cauchy-Kowalewskaya procedure as building block of high-order finite volume solvers for hyperbolic balance laws***

**Abstract:** The numerical solutions partial differential equations is required for several areas of applied mathematics. A general approach for solving the wide range of partial differential equations is not available and each family of problems demands for a family of suitable frameworks where solutions can be obtained. In this sense hyperbolic balance laws, which govern several transport phenomena among others, are of interest in this research, particularly, the framework of finite volume methods. Finite volume schemes are completely defined once numerical fluxes, jump discontinuities and source terms are set. High order of accuracy in space and time, in globally explicit schemes are achieved once local predictor within finite computational cells are provided for evaluating the numerical fluxes and source terms operators. In this work we are interested on the family of ADER finite volume methods, where local predictors consisting of Taylor series expansion in time where the time derivatives are computed from the well known Cauchy-Kowalewskaya procedure, [1]. This is a very cumbersome procedure where time-derivatives are completely obtained from purely spatial derivatives. This offers practical advantages since the spatial derivatives can be systematically obtained from governing equations of interest itself, [1, 2]. In this work, we present a simplified procedure to obtain time derivatives where not only spatial derivatives of the data are used but also the derivative in space and time of Jacobian matrices of the flux and source terms. This new approach requires a strategy for getting approximations of gradient of Jacobian matrices without using the gradients of the data. We observe that using interpolation on a suitable space-time nodal distribution, the

expected theoretical orders of accuracy are achieved. [1] E. F. Toro and V. A. Titarev. Solution of the generalised Riemann problem for advection–reaction equations. Proceedings of the Royal Society of London A, 458:271–281, 2002. [2] E. F. Toro and G. I. Montecinos. Implicit, semi-analytical solution of the generalized riemann problem for stiff hyperbolic balance laws. Journal of Computational Physics, 303:146–172, 2015.

Joaquin Mura  
Universidad Técnica Federico Santa María, Chile

***On two Inverse Problems in biomechanics***