Nonlinear Elliptic PDEs at the End of the World

Punta Arenas, March 2 – 6 , 2015

Program

March 2 – 6, 2015

	Monday 2	Tuesday 3	Wednesday 4	Thursday 5	Friday 6
8:20-9:00	Opening				
9:00-9:40	H. Berestycki	N.E. Dancer	S. Terracini	H. Ishii	C. Cortázar
9:50-10:30	B. Ruf	A. Pistoia	M. Clapp	I. Birindelli	P. Esposito
10:30-11:00	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
11:00-11:40	M. Grossi	Y. Sire	F. Pacella	F. Leoni	P. Felmer
11:50-12:30	M. del Pino	M. Kowalczyk	P. Piccione	L. Rossi	P.H Rabinowitz
12:40:13:20	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
13:20-14:30	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
14:30-15:10	M. Sáez	B. Sirakov		F. Gladiali	
15:20-16:00	T. D'Aprile	D. Bonheure		S. Martínez	
16:00-16:30	*Coffee break*	*Coffee break*		*Coffee break*	
16:30-17:10	G. Vaira	I. Ianni		16:30-16:55 O. Agudelo	
	17:20-18:00 I. Guerra	17:15-17:40 S. Kim		17:00-17:25 L. López	
		17:45-18:10 C. Román		17:30-17:55 J. Zhang	
				18:00-18:25 E. Topp	
18:30	Welcome Cocktail				

Abstracts

Catenoids, Liouville euqation and the Allen-Cahn equation

Oscar Agudelo

University of West Bohemia in Plzen
oiagudel@ntis.zcu.cz

In this work we present a review on existence of new families of axially symmetric entire solutions to the Allen-Cahn equation $\Delta u + u(1-u^2) = 0$ in $\mathbb{R}^N$ for $N \geq 3$. We exhibit a strong connection with the theory of minimal surfaces and the Liouville equation. We give precise information on the asymptotic behavior of the solutions and their Morse Index. This is a joint work with Manuel del Pino from the University of Chile and Jung Cheng Wei from the University of British Columbia.

The effect of domain shape on reaction-diffusion equations

Henri Berestycki

EHESS Paris
h.beres@gmail.com

I will discuss some reaction-diffusion equations motivated by biology and medicine of bistable type. The aim is to understand the effect of the shape of the domain on propagation or on blocking of advancing waves. I will first describe the motivations of these questions and present a result about the existence of generalized transition waves. I will then discuss various geometric conditions that lead to either blocking, or partial propagation, or complete propagation. These questions involve new qualitative results for some non-linear elliptic and parabolic partial differential equations. I report here on joint work with Juliette Bouhours and Guillemette Chapuisat.

TBA

Isabeau Birindelli

Sapienza Università di Roma
isabeau@mat.uniroma1.it

Semi-classical bound states for NLS : concentration on circles

Denis Bonheure

Université libre de Bruxelles
denis.bonheure@ulb.ac.be

In Quantum Mechanics, the nonlinear Schrödinger equation (NLS) with a magnetic field B, having source in A, and a scalar (electric) potential U has the form

$$i\hbar\frac{\partial\psi}{\partial t} = (i\hbar\nabla + A(x))^2\,\psi + U(x)\psi = f(|\psi|^2)\psi, \quad x \in \mathbb{R}^3$$

Many efforts in the past years have been devoted to the study of semi-classical standing waves, namely solutions of the form $\psi(x,t) = e^{-\frac{E}{\hbar}t}\,u(x)$ assuming $\hbar$ is small. When $A \equiv 0$, the existence of solutions concentrating around some point (or a set of points) or more generally around a manifold, has been proved by elaborated methods.

When $A \not\equiv 0$, it has been shown by Kurata, Cingolani and Cingolani-Secchi that the magnetic field does not play any role for solutions that concentrate around a single point or around multiple points. Namely, the points where the concentration occurs are not determined by the magnetic potential and their location only depends on the critical points of the electric potential.

In this talk, I will discuss a new class of solutions in the case $A \not\equiv 0$. Namely, solutions that concentrate on circles. The new feature is that the position of the circle depends on both the electric and the magnetic potentials.

The talk is based on a joint work with J. Di Cosmo & J. Van Schaftingen and a joint work with S. Cingolani & M. Nys.

Groundstates of critical and supercritical problems of Brezis-Nirenberg type

Monica Clapp

Universidad Nacional Autónoma de México
monica.clapp@im.unam.mx

We study the existence of rotationally-invariant ground states to supercritical problems of the form

$$-\Delta v = \lambda v + |v|^{p-2}\,v \;\text{ in } \Omega, \qquad v = 0 \text{ on } \partial\Omega,$$

in an $O(k+1)$-invariant domain Ω in $\mathbb{R}^N$, $1 \le k \le N-3$, at the $(k+1)$-st critical exponent $p = \frac{2(N-k)}{N-k-2}$, for any $\lambda \in \mathbb{R}$. We show that $O(k+1)$-invariant ground states exist for λ in some interval to the left hand side of each $O(k+1)$-eigenvalue, and that no $O(k+1)$-invariant ground states exist in some interval $(-\infty, \lambda_*)$ with $\lambda_* > 0$ if $k \geq 2$.

This question is related to the existence of ground states to the anisotropic critical problem

$$-\text{div}(a(x)\nabla u) = \lambda b(x)u + c(x)\,|u|^{2^*-2}\,u \quad \text{in } \Theta, \qquad u = 0 \quad \text{on } \partial\Theta,$$

where a, b, c are positive continuous functions on $\overline{\Theta}$. We give a minimax characterization for the ground states of this problem, study the ground state energy level as a function of λ, and obtain a bifurcation result for ground states.

This is joint work with Angela Pistoia and Andrzej Szulkin.

Nonlocal diffusion in exterior domains

Carmen Cortázar

Pontificia Universidad Católica de Chile
ccortaza@mat.puc.cl

We study the large-time behavior of solutions to a non-local diffusion equation, $u_t = J * u - u := Lu$, in an exterior domain, Ω, which excludes one or several holes, and with zero Dirichlet data on $\mathbb{R}^N \setminus \Omega$. J is assumed to be a nonnegative continuous function with unit integral. In this study the stationary solution plays a fundamental role.

This is a joint work with Manuel Elgueta, Fernando Quirós and Noemi Wolanski.

Blowing-up solutions for the singular Liouville equation on closed surfaces

Teresa D'Aprile

Università di Roma "Tor Vergata"
daprile@mat.uniroma2.it

Let (Σ, g) be a compact surface without boundary endowed with metric g. We are concerned with the existence of blowing-up solutions when the parameter ρ approaches the critical values $8\pi\mathbb{N}$ for the following singular Liouville equation:

$$-\Delta_g u = \rho\left(\frac{h(x)e^u}{\int_\Sigma h(x)e^u\,dV_g} - \frac{1}{|\Sigma|}\right) - 4\pi\sum_{i=1}^{\ell}\alpha_i\left(\delta_{p_i} - \frac{1}{|\Sigma|}\right),$$

where $\rho > 0$, $h : \Sigma \to \mathbb{R}$ is a smooth positive function, the points $p_i \in \Sigma$ are the singular sources with weights $\alpha_i > 0$. Here δ_p denotes the Dirac mass measure supported at p and $|\Sigma|$ is the area of Σ.

In particular, by employing a min-max scheme jointly with a finite dimensional reduction method, we construct solutions exhibiting a *blow-up* behavior near a finitely many number of points of Σ. We then discuss how new existence results may be deduced in a perturbative regime for the case of the sphere.

This is joint work with P. Esposito (Rome Tre University).

Some systems with large interaction

E. Norman Dancer

University of Sydney
norman.dancer@sydney.edu.au

We discuss non-negative solutions of the system

$$\begin{cases} -\Delta u = u(a - v) & \text{in } D \\ -\Delta v = v(d - u) & \text{in } D \\ u = v = 0 & \text{on } \partial D \end{cases}$$

Here D is a smooth bounded domain.

This is a limit problem for a problem in mathematical biology (in particular population studies). We also discuss the related problem where we replace the v in the first equation by its square and replace the u in the second equation by its square. This is a limit problem for condensed matter problems.

We discuss the similarities and the differences of the two equations. In particular,we discuss the multiplicity of solutions.

Green's function and infinite time bubbling in the semilinear heat equation at the critical Sobolev exponent

Manuel del Pino

Universidad de Chile
delpino@dim.uchile.cl

We discuss some new results on globally defined in time positive solutions of the semilinear heat equation with critical power nonlinearity and Dirichlet boundary conditions in a bounded domain. For any given number k we can find a solution that, as time grows, blows up exactly at k points of the domain with a bubbling profile that can be precisely computed. This is joint work with Carmen Cortázar and Monica Musso.

On a quasilinear mean field equation with an exponential nonlinearity

Pierpaolo Esposito

Università di Roma 3
esposito@mat.uniroma3.it

We will discuss a mean field equation involving the N-Laplace operator and an exponential nonlinearity in dimension $N \geq 2$ on bounded domains with homogeneous Dirichlet boundary condition. A quantization property is derived in the non-compact case by a detailed asymptotic analysis, yielding to the compactness of the solutions set in the so-called non-resonant regime. In such a regime, we will also discuss an existence result by a variational approach.

Some symmetry results for fractional semi-linear elliptic problems

Patricio Felmer

Universidad de Chile
pfelmer@dim.uchile.cl

We report about symmetry results for positive solutions for some fractional elliptic equations in a bounded domain and in $\mathbb{R}^N$. We use rearrangements for mountain pass solutions and the moving planes for general solutions.

The results are joint work with Csar Torres and Jiang Wang, respectively.

Multiplicity results for sign changing bound state solutions of a quasilinear equation

Carmen Cortázar, Marta García-Huidobro, and Pilar Herreros

Pontificia Universidad Católica de Chile
mgarcia@mat.puc.cl

In this talk we give conditions on the nonlineary f so that the problem

$$\begin{aligned} \Delta u + f(u) = 0, \quad x \in \mathbb{R}^N, N \geq 2, \\ \lim_{|x| \to \infty} u(x) = 0, \end{aligned} \tag{1}$$

has at least two solutions having a prescribed number of nodal regions and for which $u(0) > 0$. Any nonconstant solution to (1) is called a bound state solution. Bound state solutions such that $u(x) > 0$ for all $x \in \mathbb{R}^N$, are referred to as a first bound state solution, or a ground state solution. The existence of ground states for (1) has been established by many authors under different regularity and growth assumptions on the nonlinearity f, both for the Laplacian operator and the degenerate Laplacian operator, see for example [1, 2], [3] and [6] in the case of a regular f ($f \in C[0, \infty)$) for the case of the semilinear equation, and [7], [8] and [5] for both the singular and regular case in the quasilinear situation. The main assumptions on the nonlinearity f are

(f_1) f is a continuous function defined in $\mathbb{R}$, and f is locally Lipschitz in $\mathbb{R} \setminus \{0\}$.

(f_2) There exists $\delta > 0$ such that if we set $F(s) = \int_0^s f(t)dt$, it holds that $F(s) < 0$ for all $0 < |s| < \delta$, and $\lim\limits_{s \to -\infty} F(s) = \lim\limits_{s \to \infty} F(s)$, $F(s) < \lim\limits_{s \to \infty} F(s)$ for all $s \in \mathbb{R}$.

(f_3) F has a local maximum at some $\gamma \in (\delta, \infty)$ and $F(\gamma) > 0$.

(f_4) there exists $\theta \in (0, 1)$ such that

$$\lim_{s \to \infty} \Big(\inf_{s_1, s_2 \in [\theta s, s]} Q(s_2) \Big(\frac{s}{f(s_1)} \Big)^{N/2} \Big) = \infty, \tag{2}$$

where $Q(s) := 2NF(s) - (N-2)sf(s)$.

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Some nonradial bifurcation results for the Hardy problem

Francesca Gladiali

Università di Sassari
fgladiali@uniss.it

I will consider the Hardy problem

$$\begin{cases} -\Delta u - \frac{l}{|x|^2} u = u^p & \text{in } \Omega \\ u \geq 0 & \text{in } \Omega \\ u \in H_0^1(\Omega), \end{cases} \tag{1}$$

where $\Omega = \mathbb{R}^N$ or $\Omega = B_1$, $N \geq 3$, $p > 1$ and $l < \frac{(N-2)^2}{4}$. Using a suitable map we transform the problem (1) into a another one without the singularity $\frac{1}{|x|^2}$. Then we obtain infinitely many nonradial bifurcation points corresponding to some explicit values of l.

The 2x2 Toda System in a general setting

Massimo Grossi

Sapienza Università di Roma
massimo.grossi@uniroma1.it

We consider the classical 2×2 Toda system where the classical Cartan matrix is replaced by a more general one. Using the bifurcation theory we derive the existence of a nontrivial pair of radial solutions. The Legendre polynomials plays a crucial role in the costruction of the solution.

This is a joint paper with F. Gladiali and J. Wei.

Multiplicity of solutions for a nearly critical elliptic equation in a bounded domain in $\mathbb{R}^3$

Ignacio Guerra

Universidad de Santiago de Chile
ignacio.guerra@usach.cl

Let Ω be a smooth bounded domain in $\mathbb{R}^3$. We consider the following Dirichlet boundary value problem

$$\begin{aligned} -\Delta u = u^{5-\varepsilon} + \lambda u^q, \quad u > 0 \quad \text{in} \quad \Omega, \\ u = 0 \qquad\qquad \text{on} \quad \partial\Omega, \end{aligned}$$

where $1 < q < 3$, $\lambda > 0$, and $\varepsilon > 0$. We show that in suitable ranges for the parameters λ and ε, this problem has at least two solutions. Additionally if $2 \leq q < 3$, we prove the existence of at least three solutions.

This work is in collaboration with Wenjing Chen (Southwest University, Chongqing, China).

Asymptotic analysis and sign-changing bubble towers for Lane-Emden problems

Isabella Ianni

Università di Napoli 2
isabella.ianni@unina2.it

Let Ω be a smooth bounded domain of $\mathbb{R}^2$. We consider the semilinear Lane-Emden problem

$$\begin{cases} -\Delta u = |u|^{p-1}u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \qquad (\mathcal{E}_p)$$

where $p > 1$. We analyze the asymptotic behavior of sequences of solutions $(u_p)_p$ of $(\mathcal{E}_p)$ when the exponent p of the nonlinearity tends to infinity. The results are obtained in collaboration with F. De Marchis (Roma Tor Vergata) and F. Pacella (Roma Sapienza).

Metastability for parabolic equations with drift

Hitoshi Ishii

Waseda University
hitoshi.ishii@waseda.jp

I will outline pde methods analyzing the exponentially long time behavior of solutions to linear uniformly parabolic equations which are small perturbations of a transport equation with vector field having a globally stable point. The results say that the solutions converge to a constant, which is either the initial value at the stable point or the boundary value at the minimum of the associated quasi-potential, which are due to Freidlin and Wentzell and Freidlin and Koralov, and applies also to semilinear parabolic equations. This is based on a joint work with Takis Souganidis of the University of Chicago.

A non-compactness result on the fractional Yamabe problem in large dimensions

Seunghyeok Kim

Pontificia Universidad Católica de Chile
shkim0401@gmail.com

Let (X^{n+1}, g^+) be an $(n+1)$-dimensional asymptotically hyperbolic manifold with a conformal infinity $(M^n, [\hat{h}])$. The aim of this talk is to discuss the compactness issue on the fractional Yamabe equation

$$P^\gamma[g^+, \hat{h}](u) = u^{\frac{n+2\gamma}{n-2\gamma}}, \quad u > 0 \quad \text{on } M$$

where $P^\gamma[g^+, \hat{h}]$ is the fractional conformal Laplacian whose leading term is $(-\Delta)^\gamma$. Precisely, we show that there is an asymptotically hyperbolic metric g^+ on the half space $X = \mathbb{R}^{n+1}_+$, which is conformally equivalent to the unit ball B^{n+1}, for which the solution set of the fractional Yamabe equation is non-compact provided that the dimension n is sufficiently high. We will also observe that the optimal dimension n which gives rise to blow-up solutions depends on $\gamma \in (0,1)$. This is a joint work with Monica Musso (PUC, Chile) and Juncheng Wei (UBC, Canada).

Kink dynamics in the ϕ^4 model: asymptotic stability in the odd space

Michal Kowalczyk

Universidad de Chile
kowalczy@dim.uchile.cl

In this talk I will discuss the following nonlinear wave equation

$$\phi_{tt} - \phi_{xx} = \phi(1 - \phi^2),$$

which is known as the one dimensional ϕ^4 model. The main result I will describe is the asymptotic stability of the kink $H(x) = \tanh(\frac{x}{\sqrt{2}})$ in the space of odd functions.

Local estimates for fully nonlinear elliptic differential inequalities and applications

Fabiana Leoni

Sapienza Università di Roma
leoni@mat.uniroma1.it

We present some local estimates for viscosity sub and supersolutions of fully nonlinear uniformly elliptic equations with lower order terms. We further discuss some apllications and consequences, from weak Harnack in- equalities to non existence Liouville type theorems.

Bubbling solutions for nonlocal elliptic problems

Luis López Ríos

Universidad de Buenos Aires
llopez@dm.uba.ar

We investigate bubbling solutions for the nonlocal equation

$$\mathrm{A}_\Omega^s u = u^p, u > 0 \text{ in } \Omega,$$

under homogeneous Dirichlet conditions, where Ω is a bounded and smooth domain. The operator A_Ω^s, $s \in (0,1)$, stands for two types of nonlocal operators: the spectral fractional Laplacian and the restricted fractional Laplacian. We construct solutions when the exponent $p = (n+2s)/(n-2s) \pm \varepsilon$ is close to the critical one, concentrating as $\varepsilon \to 0^+$ near critical points of a reduced function involving the Green and Robin functions of the domain. We provide also in both sub and supercritical case a precise asymptotic profile of the blow up of these solutions as $\varepsilon \to 0^+$. This is a joint work with Y. Sire (Marseille) and J. Davila (Santiago).

Integrable steady states for a nonlocal equation

Salomé Martínez

Universidad de Chile
samartin@dim.uchile.cl

We consider the following nonlocal equation

$$\int J\left(\frac{x-y}{g(y)}\right)\frac{u(y)}{g(y)}dy - u(x) = 0 \qquad x \in \mathbb{R},$$

where J is an even, compactly supported, Hölder continuous kernel with unit integral and g is a continuous positive function. Our main concern will be with unbounded functions g. More precisely, we study the influence of the growth of g at infinity on the integrability of positive solutions of this equation, therefore determining the asymptotic behavior as $t \to +\infty$ of the solutions to the associated evolution problem in terms of the growth of g.

Symmetry of solutions of elliptic systems

Filomena Pacella

Sapienza Università di Roma
pacella@mat.uniroma1.it

We present some results about partial symmetry of solutions of some nonlinear elliptic systems in rotationally invariant domains. The method used relies strongly on maximum principles and on the analysis of the linearized system at a solution. The systems considered are cooperative, but not necessarily of gradient type, in bounded or unbounded domains. Several related questions and open problems will be presented. The results have been obtained in collaboration with L. Damascelli and F. Gladiali

Multiple solutions of the singular Yamabe problem on spheres via topological and analytical techniques

Paolo Piccione

Universidade de Sao Paulo
piccione.p@gmail.com

I will discuss a topological proof of the existence of infinitely many complete metrics in $S^m \setminus S^k$, $m \geq 5$ and $1 \leq k < \frac{m-2}{2}$ having constant scalar curvature, and conformal to the round metric. For the case $k = 1$, I will show how to use bifurcation theory to obtain the existence of uncountably many branches of solutions of the problem.

Toda system: degree and blow-up

Angela Pistoia

Sapienza Università di Roma
pistoia@dmmm.uniroma1.it

We prove existence of continua of solutions to a SU(3) Toda system which exhibit partial blow-up or asymmetric blow-up. The results have been obtained in collaboration with Teresa D'Aprile and David Ruiz.

Solution patterns for an Allen-Cahn model problem

Paul H. Rabinowitz

University of Wisconsin-Madison
rabinowi@math.wisc.edu

We will survey recent work with Jaeyoung Byeon on the structure of the set of solutions of an Allen-Cahn model for phase transitions. The simplest solutions are obtained as local minimizers of a corresponding energy functional. More complex solutions are obtained with the aid of heat flow arguments in invariant regions.

Large conformal metrics with prescribed sign-changing Gauss curvature

Carlos Román

Université Pierre et Marie Curie
I@ann.jussieu.fr

Let (M, g) be a two dimensional compact Riemannian manifold of genus $g(M) > 1$. Let f be a smooth function on M such that

$$f \geq 0, \quad f \not\equiv 0, \quad \min_M f = 0.$$

Let $p_1, ..., p_n$ be any set of points at which $f(p_i) = 0$ and $D^2 f(p_i)$ is non-singular. We prove that for all sufficiently small $\lambda > 0$ there exists a family of "bubbling" conformal metrics $g_\lambda = e^{u_\lambda} g$ such that their Gauss curvature is given by the sign-changing function $K_{g_\lambda} = -f + \lambda^2$. Moreover, the family u_λ satisfies

$$u_\lambda(p_j) = -4 \log \lambda - 2 \log \left(\frac{1}{\sqrt{2}} \log \frac{1}{\lambda} \right) + O(1)$$

and

$$\lambda^2 e^{u_\lambda} \rightharpoonup 8\pi \sum_{i=1}^{n} \delta_{p_i}, \qquad \text{as } \lambda \to 0,$$

where δ_p designates Dirac mass at the point p. This is joint work with Manuel del Pino.

Extensions of Freidlin-Gartner's formula to general reaction terms

Luca Rossi

Università di Padova
lucar@math.unipd.it

The Freidlin-Gartner formula expresses the asymptotic speed of spreading for spatial-periodic Fisher-KPP equations in terms of the principal eigenvalues of a family of linear operators. One cannot expect the same formula to hold true for the other classes of reaction terms (monostable, combustion, bistable). However, these eigenvalues have been later related with the minimal speeds of pulsating travelling fronts, yielding a formula for the spreading speed which is not unreasonable to expect to hold for any reaction term. We will see that it is indeed the case. The method presented applies to equations whose terms depend arbitrarily on time and space, highlighting a general connection between the asymptotic speed of spreading and almost planar transition fronts.

On a heat equation with exponential nonlinearity in $\mathbb{R}^2$

Bernhard Ruf

Università degli Studi di Milano
ruf@mat.unimi.it

We consider a semilinear heat equation with exponential nonlinearity in $\mathbb{R}^2$. We prove that local solutions do not exist for certain data in the Orlicz space $\exp L^2(\mathbb{R}^2)$, even though a small data global existence result holds in the same space $\exp L^2(\mathbb{R}^2)$. Moreover, some suitable subclass of $\exp L^2(\mathbb{R}^2)$ for local existence and uniqueness is proposed.

Fractional mean curvature flow

Mariel Sáez

Pontificia Universidad Católica de Chile
msaezt@mat.puc.cl

In this work we study the fractional analog to the classical mean curvature flow. Namely, we consider the evolution of surfaces with normal speed equal to the fractional mean curvature and analyze their behavior under suitable assumptions. Finally we contrast our results with the behavior of surfaces evolving under classical mean curvature flow. This is joint work with Enrico Valdinoci.

Proportionality of solutions of nonlinear elliptic systems

Boyan Sirakov

Pontificia Universidade Catlica do Rio de Janeiro
bsirakov@mat.puc-rio.br

We study a class of systems of nonlinear elliptic equations which include Lotka-Volterra models, models from the theory of Bose-Einstein condensates, and models of chemical reactions. We introduce a method for proving proportionality of components of the system set in an unbounded domain, thus reducing it to a single equation.

Nonlocal self-improving properties

Yannick Sire

Université Aix-Marseille
sire@cmi.univ-mrs.fr

I will describe a regularity result for nonlocal equations which states that any weak solution is actually in a better space both in the differentiability and integrability scales. This is a completely nonlocal phenomenon and a somehow nonlocal counterpart of results by Meyers. This is joint work with T. Kuusi and G. Mingione.

Existence and regularity of solutions to optimal partition problems involving Laplacian eigenvalues

Miguel Ramos, Hugo Tavares and Susanna Terracini

Università di Torino
susanna.terracini@unito.it

Let $\Omega \subset \mathbb{R}^N$ be an open bounded domain and $m \in \mathbb{N}$. Given $k_1, \ldots, k_m \in \mathbb{N}$, we consider a wide class of optimal partition problems involving Dirichlet eigenvalues of elliptic operators, including the following

$$\inf\left\{\Phi(\omega_1,\ldots,\omega_m) := \sum_{i=1}^{m} \lambda_{k_i}(\omega_i) :\ (\omega_1,\ldots,\omega_m) \in \mathcal{P}_m(\Omega)\right\},$$

where $\lambda_{k_i}(\omega_i)$ denotes the k_i–th eigenvalue of $(-\Delta, H_0^1(\omega_i))$ counting multiplicities, and $\mathcal{P}_m(\Omega)$ is the set of all open partitions of Ω, namely

$$\mathcal{P}_m(\Omega) = \{(\omega_1, \ldots, \omega_m) : \ \omega_i \subset \Omega \text{ open}, \ \omega_i \cap \omega_j = \emptyset \ \forall i \neq j\}.$$

We prove the existence of an open optimal partition $(\omega_1, \ldots, \omega_m)$, proving as well its regularity in the sense that the free boundary $\cup_{i=1}^m \partial\omega_i \cap \Omega$ is, up to a residual set, locally a $C^{1,\alpha}$ hypersurface.

In order to prove this result, we first treat some general optimal partition problems involving all eigenvalues up to a certain order. This class of problems includes the one with cost function $\Phi_p(\omega_1, \ldots, \omega_m) := \sum_{i=1}^m \left(\sum_{j=1}^{k_i} (\lambda_j(\omega_i))^p\right)^{1/p}$, whose solutions approach the solutions of the original problem as $p \to +\infty$. The study of this new class of problems is done via a singular perturbation approach with a class of Schrödinger-type systems which models competition between different groups of possibly sign-changing components. An optimal partition appears in relation with the nodal set of the limiting components, as the competition parameter becomes large.

Regularity results and large time behavior for integro-differential equations with coercive hamiltonians

Erwin Topp

Universidad Técnica Federico Santa María
emtopp@gmail.com

In this paper we obtain regularity results for elliptic integro-differential equations driven by the stronger effect of coercive gradient terms. This feature allows us to construct suitable strict supersolutions from which we conclude Hölder estimates for bounded subsolutions. In many interesting situations, this gives way to a priori estimates for subsolutions. We apply this regularity results to obtain the ergodic asymptotic behavior of the associated evolution problem in the case of superlinear equations. One of the surprising features in our proof is that it avoids the key ingredient which are usually necessary to use the Strong Maximum Principle: linearization based on the Lipschitz regularity of the solution of the ergodic problem. The proof entirely relies on the Hölder regularity.

This is a joint work with Guy Barles, Olivier Ley and Shigeaki Koike.

Sign-changing solutions for the Brezis-Nirenberg problem

Giusi Vaira

Sapienza Università di Roma
giusi.vaira@gmail.com

We deal with the Brezis-Nirenberg problem, namely with the problem

$$\begin{cases} -\Delta u = \lambda u + |u|^{p-1}u & \text{in } \Omega \\ \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{3}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$, $N \geq 3$, λ is a positive and real parameter, while $p + 1 = \frac{2N}{N-2}$ is the critical exponent for the embedding of $H_0^1(\Omega)$ into $L^{p+1}(\Omega)$.
We show some recent existence result on sign-changing solutions for the problem (3) in low dimensions and in higher dimensions.

Sign-changing Solutions of Nonlinear Schrödinger-Poisson Systems in $\mathbb{R}^3$

Jianjun Zhang

Universidade Federal da Paraiba
zhangjianjun09@tsinghua.org.cn

We discuss the sign-changing solutions of the Schrödinger-Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u = f(u) & \text{in } \mathbb{R}^3, \\ -\Delta \phi = u^2 & \text{in } \mathbb{R}^3. \end{cases}$$

By using the method of invariant sets of descending flow, we prove that this system has multiple sign-changing solutions. In particular, the nonlinear term includes the power-type nonlinearity $f(u) = |u|^{p-2}u$ for the well-studied case $p \in (4, 6)$ and the less studied case $p \in (2, 4)$, and for the latter case few existence results are available in the literature.

This is joint work with Z.-Q. Wang and Zhaoli Liu.