Blowing-up solutions for the singular Liouville equation on closed surfaces

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Let (Σ, g) be a compact surface without boundary endowed with metric g. We are concerned with the existence of blowing-up solutions when the parameter ρ approaches the critical values $8\pi\mathbb{N}$ for the following singular Liouville equation:

$$-\Delta_g u = \rho\left(\frac{h(x)e^u}{\int_\Sigma h(x)e^u\, dV_g} - \frac{1}{|\Sigma|}\right) - 4\pi \sum_{i=1}^{\ell} \alpha_i \left(\delta_{p_i} - \frac{1}{|\Sigma|}\right),$$

where $\rho > 0$, $h : \Sigma \to \mathbb{R}$ is a smooth positive function, the points $p_i \in \Sigma$ are the singular sources with weights $\alpha_i > 0$. Here δ_p denotes the Dirac mass measure supported at p and $|\Sigma|$ is the area of Σ.

In particular, by employing a min-max scheme jointly with a finite dimensional reduction method, we construct solutions exhibiting a *blow-up* behavior near a finitely many number of points of Σ. We then discuss how new existence results may be deduced in a perturbative regime for the case of the sphere.

This is joint work with P. Esposito (Rome Tre University).