

Multiplicity results for sign changing bound state solutions of a quasilinear equation

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In this talk we give conditions on the nonlinearity f so that the problem

$$\begin{aligned} \Delta u + f(u) &= 0, \quad x \in \mathbb{R}^N, N \geq 2, \\ \lim_{|x| \rightarrow \infty} u(x) &= 0, \end{aligned} \tag{1}$$

has at least two solutions having a prescribed number of nodal regions and for which $u(0) > 0$. Any nonconstant solution to (1) is called a bound state solution. Bound state solutions such that $u(x) > 0$ for all $x \in \mathbb{R}^N$, are referred to as a first bound state solution, or a ground state solution. The existence of ground states for (1) has been established by many authors under different regularity and growth assumptions on the nonlinearity f , both for the Laplacian operator and the degenerate Laplacian operator, see for example [1, 2], [3] and [6] in the case of a regular f ($f \in C[0, \infty)$) for the case of the semilinear equation, and [7], [8] and [5] for both the singular and regular case in the quasilinear situation. The main assumptions on the nonlinearity f are

(f_1) f is a continuous function defined in \mathbb{R} , and f is locally Lipschitz in $\mathbb{R} \setminus \{0\}$.

(f_2) There exists $\delta > 0$ such that if we set $F(s) = \int_0^s f(t)dt$, it holds that $F(s) < 0$ for all $0 < |s| < \delta$, and $\lim_{s \rightarrow -\infty} F(s) = \lim_{s \rightarrow \infty} F(s)$, $F(s) < \lim_{s \rightarrow \infty} F(s)$ for all $s \in \mathbb{R}$.

(f_3) F has a local maximum at some $\gamma \in (\delta, \infty)$ and $F(\gamma) > 0$.

(f_4) there exists $\theta \in (0, 1)$ such that

$$\lim_{s \rightarrow \infty} \left(\inf_{s_1, s_2 \in [\theta s, s]} Q(s_2) \left(\frac{s}{f(s_1)} \right)^{N/2} \right) = \infty, \tag{2}$$

where $Q(s) := 2NF(s) - (N-2)sf(s)$.

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