

Optimization of Energy Production and Transport

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Approaches by Decomposition

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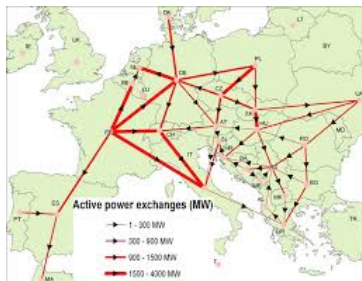
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Motivation

An energy **production and transport optimization problem** on a grid modeling energy exchange across countries.²



- Stochastic dynamical problem.
- Discrete time formulation (one-day time step).
- Large-scale problem (many countries).

²But the framework remains valid for smaller energy management problems.

Goal

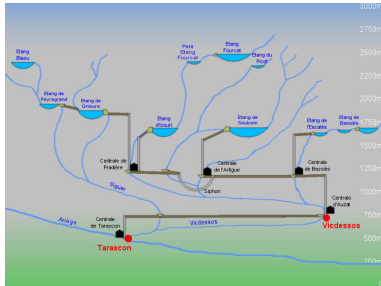
Obtain **cost-to-go functions** for a **large scale** stochastic optimal control problem in discrete time.

- In order to obtain **decision strategies** (closed-loop controls), we have to use **dynamic programming** or related methods.
 - **Assumption**: Markovian case,
 - **Difficulty**: **curse of dimensionality**.
- To overcome the barrier of the dimension we want to use **decomposition/coordination** techniques, which makes it difficult to take into account the **information pattern** induced by the stochasticity in the optimization problem.

This is a part of a broader project, aiming to develop decision analysis tools for long-term investment problems.

Previous work

We studied the application of stochastic decomposition to the optimization of an hydraulic valley.



Valley: a **tree structure** with

- **nodes**: dams
- **arcs**: flows

We solved this problem using a **price-decomposition** approach (see [Carpentier et al, 2017]).

We want to extend this work in two directions:

- more complex topologies (**graphs** rather than **trees**)
- other decomposition algorithms (**allocation**, **prediction**).

Lecture outline

1 Introduction

- The production and transport problem
- Mixing decomposition and dynamic programming

2 Decomposition methods

- Price decomposition
- Resource allocation
- Interaction prediction

3 Discussion

1 Introduction

- The production and transport problem
- Mixing decomposition and dynamic programming

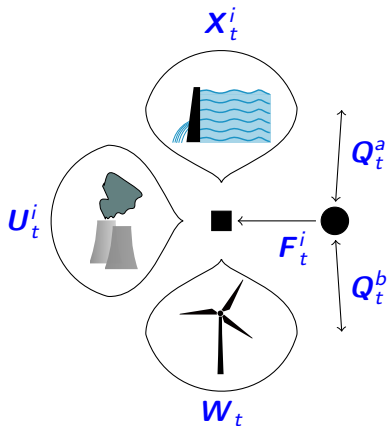
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Production at each node of the grid

At each node i of the grid, we formulate a **production problem** on a discrete time horizon $\llbracket 0, T \rrbracket$, involving the following variables at each time t :



- X_t^i : **state variable**
(dam level)
- U_t^i : **control variable**
(units production)
- F_t^i : **grid flow**
(import/export from the grid)
- W_t : **noise**
(consumption, renewable)

The noise W_t is supposed to be shared across the different nodes.

A stochastic optimization problem decoupled in space

At **each node** i of the grid, we have to solve a stochastic optimal control subproblem depending on the grid flow process \mathbf{F}^i :³

$$J_{\mathfrak{F}}^i[\mathbf{F}^i] = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E} \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right),$$

s.t. $\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1})$,

$\mathbf{X}_t^i \in \mathcal{X}_t^{i, \text{ad}}$, $\mathbf{U}_t^i \in \mathcal{U}_t^{i, \text{ad}}$,

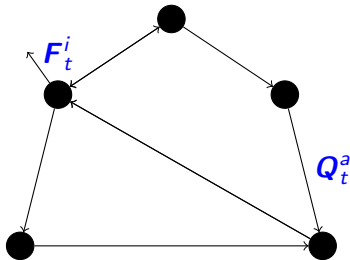
$\mathbf{U}_t^i \preceq \mathcal{F}_t$,

The last equation is the **measurability constraint**, where \mathcal{F}_t is the σ -field generated by the noises $\{\mathbf{W}_\tau\}_{\tau=1\dots t}$ up to time t .

³The notation $J_{\mathfrak{F}}^i[\cdot]$ means that the argument of $J_{\mathfrak{F}}^i$ is a *random variable*.

Modeling exchanges between countries...

The grid is represented by a **directed graph** $\mathcal{G} = (\mathcal{N}, \mathcal{A})$. At each time $t \in \llbracket 0, T - 1 \rrbracket$ we have:



- a flow Q_t^a through each arc a , inducing a cost $c_t^a(Q_t^a)$, modeling the exchange between two countries
- a grid flow F_t^i at each node i , resulting from the balance equation

$$F_t^i = \sum_{a \in \text{input}(i)} Q_t^a - \sum_{b \in \text{output}(i)} Q_t^b$$

... A transport problem decoupled in time

At each time step $t \in \llbracket 0, T - 1 \rrbracket$, we define the transport cost as the sum of the cost of the flows Q_t^a through the arcs a of the grid:

$$J_{\mathcal{X},t}[Q_t] = \mathbb{E} \left(\sum_{a \in \mathcal{A}} c_t^a(Q_t^a) \right),$$

where the c_t^a 's are easy to compute functions (say quadratic).

Kirchhof's law

The balance equation stating the conservation between Q_t and F_t rewrites in the following matrix form:

$$A Q_t + F_t = 0,$$

where A is the node-arc incidence matrix of the grid.

The overall production transport problem

The *production cost* aggregates the costs at all nodes i :

$$J_{\mathfrak{P}}[\mathbf{F}] = \sum_{i \in \mathcal{N}} J_{\mathfrak{P}}^i[\mathbf{F}^i],$$

and the *transport cost* $J_{\mathfrak{T}}$ aggregates the costs at all time t :

$$J_{\mathfrak{T}}[\mathbf{Q}] = \sum_{t=0}^{T-1} J_{\mathfrak{T},t}[\mathbf{Q}_t].$$

The compact **production-transport problem** formulation writes:

$$\begin{aligned} \min_{\mathbf{Q}, \mathbf{F}} \quad & J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \\ \text{s.t.} \quad & \mathbf{A}\mathbf{Q} + \mathbf{F} = \mathbf{0}. \end{aligned}$$

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Introducing Dual Approximate Dynamic Programming

The **Decomposition/Coordination** methods we want to deal with are iterative algorithms involving the following ingredients.

- **Decompose** the global problem in several subproblems of smaller size,
- **Coordinate** at each iteration the subproblems using either a **price** or an **allocation**,

$$AQ + \underbrace{F}_{\text{allocation}} = 0 \quad \rightsquigarrow \quad \underbrace{\lambda}_{\text{price}}$$

- Solve the subproblems using **Dynamic Programming** (when a state is available in the subproblem), taking into account the **price** or the **allocation** transmitted by the coordination.

Production subproblems induced by decomposition

The i -th production subproblem at iteration k formulates as follows.

- Price transmission case

$$\begin{aligned} \min_{\mathbf{X}^i, \mathbf{U}^i, \mathbf{F}^i} \mathbb{E} & \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t^{(k)}, \mathbf{F}_t^i \rangle + K^i(\mathbf{X}_T^i) \right), \\ \text{s.t. } \mathbf{X}_{t+1}^i &= f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}), \\ \mathbf{U}_t^i &\preceq \mathcal{F}_t. \end{aligned}$$

- Allocation transmission case

$$\begin{aligned} \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E} & \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^{i,(k)}, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right), \\ \text{s.t. } \mathbf{X}_{t+1}^i &= f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^{i,(k)}, \mathbf{W}_{t+1}), \\ \mathbf{U}_t^i &\preceq \mathcal{F}_t. \end{aligned}$$

Approximating the subproblems

In both cases, the subproblems encompass a new “noise”, that is, either a **price multiplier** $\lambda_t^{(k)}$ or a **flow allocation** $F_t^{i,(k)}$, which may be **correlated** in time. The **white noise** assumption fails.

Dynamic Programming cannot be used for solving the subproblems.

In order to overcome this difficulty, we use a **trick** that involves **approximating** the new noise (either λ_t^k or F_t^k) by its **conditional expectation** w.r.t. a chosen random variable Y_t .

Assume that the process Y has a given dynamics:

$$Y_{t+1} = h_t(Y_t, W_{t+1}).$$

If noises W_t 's are time independent, then (X_t^i, Y_t) is a valid state for the i -th subproblem and **Dynamic Programming** applies.⁴

⁴See [Barty et al, 2010] for further details.

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Price decomposition

The production and transport optimization problem writes

$$\min_{Q, F} J_{\mathfrak{P}}[F] + J_{\mathfrak{T}}[Q] \quad \text{s.t.} \quad AQ + F = 0. \quad (1)$$

The decomposition scheme consists in dualizing the constraint, and then **approximating** the multiplier λ by its conditional expectation w.r.t. Y . This **trick** leads to the following problem

$$\max_{\lambda} \min_{Q, F} J_{\mathfrak{P}}[F] + J_{\mathfrak{T}}[Q] + \langle \mathbb{E}(\lambda \mid Y), AQ + F \rangle.$$

The last problem is equivalent to the two following problems.

- ① **Restricted** dual problem

$$\max_{\lambda \preceq Y} \min_{Q, F} J_{\mathfrak{P}}[F] + J_{\mathfrak{T}}[Q] + \langle \lambda, AQ + F \rangle,$$

- ② **Relaxed** primal problem (\rightsquigarrow lower bound of (1))

$$\min_{Q, F} J_{\mathfrak{P}}[F] + J_{\mathfrak{T}}[Q] \quad \text{s.t.} \quad \mathbb{E}(AQ + F \mid Y) = 0.$$

A dual gradient-like algorithm

Applying the Uzawa algorithm to the problem

$$\max_{\lambda} \min_{Q, F} J_{\mathfrak{P}}[F] + J_{\mathfrak{T}}[Q] + \langle \mathbb{E}(\lambda \mid \mathbf{Y}), A\mathbf{Q} + \mathbf{F} \rangle,$$

leads to a decomposition between production and transport:

$$\mathbf{F}^{(k+1)} \in \arg \min_F J_{\mathfrak{P}}[F] + \langle \mu^{(k)}, F \rangle, \quad \text{Production}$$

$$\mathbf{Q}^{(k+1)} \in \arg \min_Q J_{\mathfrak{T}}[Q] + \langle \mu^{(k)}, A\mathbf{Q} \rangle, \quad \text{Transport}$$

$$\mu^{(k+1)} = \mu^{(k)} + \rho \mathbb{E}(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)} \mid \mathbf{Y}), \quad \text{Update}$$

where we use the notation $\mu^{(k)} = \mathbb{E}(\lambda^{(k)} \mid \mathbf{Y})$.

Decomposing the transport problem

The **transport** subproblem

$$\min_{\mathbf{Q}} J_{\mathcal{T}}[\mathbf{Q}] + \langle \boldsymbol{\mu}^{(k)}, A\mathbf{Q} \rangle ,$$

writes in a detailed manner

$$\min_{\mathbf{Q}} \sum_{t=0}^{T-1} \mathbb{E} \left(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) + \langle \boldsymbol{\mu}_t^{(k)}, A\mathbf{Q}_t \rangle \right) .$$

It is evidently **decomposable** in time **and** in space (arc by arc):

$$\min_{\mathbf{Q}_t^a} \mathbb{E} \left(c_t^a(\mathbf{Q}_t^a) + \langle (A^\top \boldsymbol{\mu}_t^{(k)})^a, \mathbf{Q}_t^a \rangle \right) ,$$

leading to subproblems extremely easy to solve.

Decomposing the production problem

The **production** subproblem

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + \langle \boldsymbol{\mu}^{(k)}, \mathbf{F} \rangle,$$

decomposes node by node

$$\min_{\mathbf{F}^i} J_{\mathfrak{P}}^i[\mathbf{F}^i] + \langle \boldsymbol{\mu}^{i,(k)}, \mathbf{F}^i \rangle,$$

that is,

$$\begin{aligned} \min_{\mathbf{X}^i, \mathbf{U}^i, \mathbf{F}^i} \mathbb{E} & \left(\sum_{t=0}^{T-1} \left(L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\mu}_t^{i,(k)}, \mathbf{F}_t^i \rangle \right) + K^i(\mathbf{X}_T^i) \right) \\ \text{s.t. } & \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) \\ & \mathbf{U}_t^i \preceq \mathcal{F}_t. \end{aligned}$$

Solving the production subproblems by DP

As $\mu_t^{(k)} \preceq \mathbf{Y}_t$, we write it as a functional $\mu_t^{(k)} = \phi_t^{(k)}(\mathbf{Y}_t)$.

We have assumed that

- the process \mathbf{W} is a white noise,
- the process \mathbf{Y} follows a dynamics $\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$,

so that **Dynamic Programming** applies for production subproblems:

$$V_T^i(x, y) = K^i(x)$$

$$V_t(x, y) = \min_{u, f} \mathbb{E} \left(L_t^i(x, u, f, \mathbf{W}_{t+1}) \right. \\ \left. + \langle \phi_t^{(k)}(y), f \rangle + V_{t+1}^i(\mathbf{X}_{t+1}^i, \mathbf{Y}_{t+1}) \right)$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = f_t^i(x, u, f, \mathbf{W}_{t+1}), \\ \mathbf{Y}_{t+1} = h_t(y, \mathbf{W}_{t+1}).$$

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Resource allocation decomposition

Resource allocation decomposition applied to the problem

$$\min_{\mathbf{Q}, \mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{I}}[\mathbf{Q}] \quad \text{s.t.} \quad \mathbf{A}\mathbf{Q} + \mathbf{F} = \mathbf{0}, \quad (2)$$

consists in rewriting the constraint $\mathbf{A}\mathbf{Q} + \mathbf{F} = \mathbf{0}$ by introducing a new variable \mathbf{V} (the **allocation**), that is,

$$\mathbf{A}\mathbf{Q} + \mathbf{V} = \mathbf{0} \quad \text{and} \quad \mathbf{F} - \mathbf{V} = \mathbf{0}.$$

Here the **trick** consists in **limiting** the measurability of variable \mathbf{V} , that is, $\mathbf{V} \preceq \mathbf{Y}$. The approximation leads to solve the following **restricted** primal problem (\rightsquigarrow **upper bound** of (2))

$$\min_{\mathbf{V} \preceq \mathbf{Y}} \left(\min_{\mathbf{F}} \left(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = \mathbf{0} \right) + \min_{\mathbf{Q}} \left(J_{\mathfrak{I}}[\mathbf{Q}] \quad \text{s.t.} \quad \mathbf{A}\mathbf{Q} + \mathbf{V} = \mathbf{0} \right) \right).$$

A primal gradient-like algorithm

Applying a gradient-like algorithm w.r.t. \mathbf{V} to the problem

$$\min_{\mathbf{V} \preceq \mathbf{Y}} \left(\min_{\mathbf{F}} \left(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \right) + \min_{\mathbf{Q}} \left(J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad \mathbf{A}\mathbf{Q} + \mathbf{V} = 0 \right) \right),$$

leads to a decomposition between production and transport:⁵

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V}^{(k)} = 0 \quad \rightsquigarrow \quad \boldsymbol{\lambda}^{(k+1)} \quad \text{Production}$$

$$\min_{\mathbf{Q}} J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad \mathbf{A}\mathbf{Q} + \mathbf{V}^{(k)} = 0 \quad \rightsquigarrow \quad \boldsymbol{\nu}^{(k+1)} \quad \text{Transport}$$

$$\mathbf{V}^{(k+1)} = \text{proj}_{\mathbf{V} \preceq \mathbf{Y}} \left(\mathbf{V}^{(k)} + \rho(\boldsymbol{\lambda}^{(k+1)} - \boldsymbol{\nu}^{(k+1)}) \right) \quad \text{Update}$$

⁵Note that we must ensure at each iteration that $\mathbf{V}_t^{(k)} \in \text{Im}\mathbf{A}$.

Decomposing the transport problem

The **transport** subproblem

$$\min_{\mathbf{Q}} J_{\mathcal{T}}[\mathbf{Q}] \quad \text{s.t.} \quad A\mathbf{Q} + \mathbf{V} = 0 ,$$

writes in a detailed manner

$$\min_{\mathbf{Q}} \sum_{t=0}^{T-1} \mathbb{E} \left(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) \right) \quad \text{s.t.} \quad A\mathbf{Q}_t + \mathbf{V}_t^{(k)} = 0 \quad \forall t .$$

It is **decomposable** in time, **but not** in space:

$$\min_{\mathbf{Q}_t} \mathbb{E} \left(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) \right) \quad \text{s.t.} \quad A\mathbf{Q}_t + \mathbf{V}_t^{(k)} = 0 .$$

The resulting subproblems are again easy to solve.

Decomposing the production problem

The **production** subproblem

$$\min_{\mathbf{F}} \left(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \right),$$

decomposes node by node

$$\min_{\mathbf{F}^i} J_{\mathfrak{P}}^i[\mathbf{F}^i] \quad \text{s.t.} \quad \mathbf{F}^i - \mathbf{V}^{i,(k)} = 0,$$

that is,

$$\begin{aligned} \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E} & \left(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{V}_t^{i,(k)}, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right), \\ \text{s.t.} \quad \mathbf{X}_{t+1}^i &= f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{V}_t^{i,(k)}, \mathbf{W}_{t+1}) \\ \mathbf{U}_t^i &\preceq \mathcal{F}_t. \end{aligned}$$

Solving the production subproblems by DP

As $\mathbf{V}_t^{i,(k)} \preceq \mathbf{Y}_t$, we write it as a functional $\mathbf{V}_t^{i,(k)} = \psi_t^{(k)}(\mathbf{Y}_t)$.

We have assumed that

- the process \mathbf{W} is a white noise,
- the process \mathbf{Y} follows a dynamics $\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$,

so that **Dynamic Programming** applies for production subproblems:

$$V_T^i(x, y) = K^i(x)$$

$$V_t(x, y) = \min_u \mathbb{E} \left(L_t^i(x, u, \psi_t^{(k)}(y), \mathbf{W}_{t+1}) + V_{t+1}^i(\mathbf{X}_{t+1}^i, \mathbf{Y}_{t+1}) \right)$$

$$\begin{aligned} \text{s.t. } \quad & \mathbf{X}_{t+1}^i = f_t^i(x, u, \psi_t^{(k)}(y), \mathbf{W}_{t+1}), \\ & \mathbf{Y}_{t+1} = h_t(y, \mathbf{W}_{t+1}). \end{aligned}$$

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Interaction Prediction

As in resource allocation, we introduce a new variable \mathbf{V} and rewrite the constraint $\mathbf{A}\mathbf{Q} + \mathbf{F} = 0$ as

$$\mathbf{A}\mathbf{Q} + \mathbf{V} = 0 \quad \text{and} \quad \mathbf{F} - \mathbf{V} = 0.$$

We again limit the measurability of variable \mathbf{V} , that is, $\mathbf{V} \preceq \mathbf{Y}$. The **interaction prediction** is in that case a mix of price decomposition and resource allocation, aiming at solving:

$$\min_{\mathbf{V} \preceq \mathbf{Y}} \max_{\mu} \left(\min_{\mathbf{F}} \left(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \right) + \min_{\mathbf{Q}} \left(J_{\mathfrak{R}}[\mathbf{Q}] + \langle \mu, \mathbf{A}\mathbf{Q} + \mathbf{V} \rangle \right) \right).$$

Note that the **constraint** is partially handled (production problem) and partially dualized (transport problem).

A fixed-point algorithm

Applying a **fixed-point algorithm** w.r.t. \mathbf{V} and $\boldsymbol{\mu}$ to the problem

$$\min_{\mathbf{V} \preceq \mathbf{Y}} \max_{\boldsymbol{\mu}} \left(\min_{\mathbf{F}} \left(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \right) + \min_{\mathbf{Q}} \left(J_{\mathfrak{T}}[\mathbf{Q}] + \langle \boldsymbol{\mu}, \mathbf{A}\mathbf{Q} + \mathbf{V} \rangle \right) \right),$$

leads to a decomposition between production and transport:

- 1 Solve the production and transport problems

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V}^{(k)} = 0 \quad \rightsquigarrow \quad \boldsymbol{\lambda}^{(k+1)} \quad \text{Production}$$

$$\min_{\mathbf{Q}} J_{\mathfrak{T}}[\mathbf{Q}] + \langle \boldsymbol{\mu}^{(k)}, \mathbf{A}\mathbf{Q} \rangle \quad \rightsquigarrow \quad \mathbf{Q}^{(k+1)} \quad \text{Transport}$$

- 2 **Update** the allocation and the multiplier:

$$\mathbf{V}^{(k+1)} = \mathbf{A}\mathbf{Q}^{(k+1)} \quad , \quad \boldsymbol{\mu}^{(k+1)} = \boldsymbol{\lambda}^{(k+1)} .$$

Decomposing the production and the transport problems

In **prediction decomposition**, the **production** subproblem is solved in the same way as in resource allocation, whereas the **transport** subproblem is solved in the same way as in price decomposition.

All that has been seen above therefore applies:

- the **production subproblem** decomposes node by node:

$$\min_{F^i} J_{\mathfrak{P}}^i[F^i] \quad \text{s.t.} \quad F^i - V^{i,(k)} = 0,$$

(Dynamic Programming applies)

- the **transport subproblem** decomposes in time and in space:

$$\min_{Q_t} \mathbb{E} \left(c_t^a(Q_t^a) + \langle (A^T \mu_t^{(k)})^a, Q_t^a \rangle \right),$$

(easy to solve subproblems)

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We want to benchmark these three methods, with:

- A **numerical** comparison, applying these algorithms to manage the European grid.
- A **theoretical** comparison: knowing that

$$\mathfrak{J}^{price} \leq \mathfrak{J}^{\#} \leq \mathfrak{J}^{resource}$$

do we have

$$\mathfrak{J}^{price} \leq \mathfrak{J}^{prediction} \leq \mathfrak{J}^{resource} \quad ?$$

- An implementation of other decomposition methods, such as **augmented Lagrangian** based methods.

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