Backward Induction in Games Without Perfect Recall

Workshop in Santiago, January 2017

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Backward Induction

The game theorists who defined the equilibrium concepts that we now think of as various forms of backwards induction, namely subgame perfect equilibrium (Selten, 1965), perfect equilibrium (Selten, 1975), sequential equilibrium (Kreps and Wilson, 1982), and quasi-perfect equilibrium (van Damme, 1984), explicitly restricted their analysis to games with perfect recall. In spite of this the concepts are well defined, exactly as they defined them, even in games without perfect recall. There is now a small literature examining the behaviour of these concepts in games without perfect recall. Jeff Kline (2005) looks at what happens in games without perfect recall to solutions defined in exactly the same way as they were defined in games with perfect recall. Joe Halpern and Rafael Pass 2016 modify the definitions of Selten and Kreps and Wilson in a somewhat different manner than we do.

BI without Perfect Recall

We shall argue that in games without perfect recall the original definitions are inappropriate. Our reading of the original papers is not that the authors were unaware that their definitions did not require the assumption of perfect recall, but rather that they were aware that without the assumption of perfect recall the definitions they gave were not the “correct” ones. In this paper we give definitions of two of these concepts, sequential equilibrium and quasi-perfect equilibrium that identify the same equilibria in games with perfect recall and behave well in games without perfect recall.

The Nature of Our Project

What we want to do is to give definitions of the “backward induction” solutions—in this paper sequential equilibrium and quasi-perfect equilibrium—that

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The Nature of Our Project

What we want to do is to give definitions of the "backward induction" solutions—in this paper sequential equilibrium and quasi-perfect equilibrium—that

- "coincides" with the original definitions in games with perfect recall, and
- satisfies the same properties in games without perfect recall as it does in games with perfect recall. In particular the inclusions that a proper equilibrium is a quasi-perfect equilibrium and a quasi-perfect equilibrium is a sequential equilibrium.

Informal Definitions

Definition 1. A player has perfect recall if, at each of his information sets he remembers what he knew and what he did in the past.

This concept was originally defined by Kuhn (1953). Later an equivalent definition was given by Selten (1975).
In games without perfect recall we distinguish between linear games (the games defined by Kuhn) and nonlinear games (an extension by Isbell (1957) and more recently discussed under the name “absent-mindedness” by Piccione and Rubinstein (1997)).
In linear games each play of the game reaches an information set at most once. For the moment we shall be restricting attention to linear games. The issues are not so different in nonlinear games and, time permitting, we shall return to discuss an extension of the concepts to nonlinear games at the end of this talk.

Informal Definitions

Definition 2. A pure strategy in an extensive form game for Player *n* is a function that maps his information sets to the set of actions such that each information set is mapped to an action available at that information set.

Definition 3. A behaviour strategy in an extensive form game for Player *n* is a function that maps his information sets to the set of probability distributions over the set of actions such that each information set is mapped to a probability distribution that puts all weight on actions available at that information set.

Definition 4. A mixed strategy in an extensive form game for Player *n* is a probability distribution over the player's pure strategies.

Pure, Behaviour, and Mixed Strategies

An immediate implication of these definitions is that the set of pure strategies is embedded in both the set of behaviour strategies and the set of mixed strategies. A pure strategy s is equivalent to the behaviour strategy that takes each information set to the probability distribution that puts weight 1 on the action that s selects at that information set. And s is equivalent to the mixed strategy that puts weight 1 on s.

Kuhn’s Theorem

Kuhn (1953) showed that if a player has perfect recall then each of his mixed strategies has an equivalent behaviour strategy, that is, a behaviour strategy that will induce the same distribution on terminal nodes as the mixed strategy, whatever the other players may choose. If the player does not have perfect recall this will not be true for all mixed strategies. Isbell (1957) showed that in a linear game each behaviour strategy of a player has an equivalent mixed strategy, though this result is almost implicit in Kuhn.

Nonexistence in Behaviour Strategies

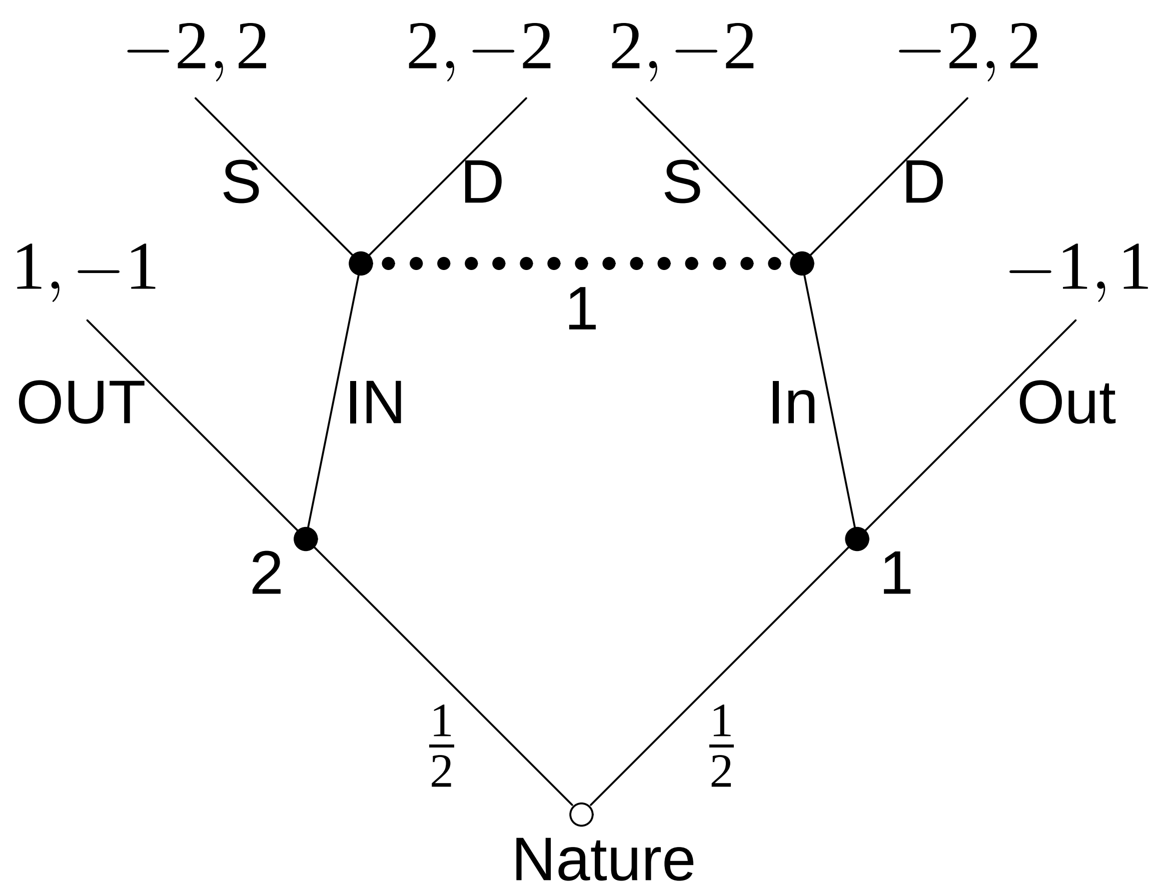
An immediate implication of the definitions of strategies is that the set of pure strategies is embedded in both the set of behaviour strategies and the set of mixed strategies. A pure strategy s is equivalent to the behaviour strategy that takes each information set to the probability distribution that puts weight 1 on the action that s selects at that information set. And s is equivalent to the mixed strategy that puts weight 1 on s.

This means that in games in which there is a unique equilibrium in mixed strategies and that mixed strategy profile is not equivalent to a profile of behaviour strategies there is no equilibrium in behaviour strategies. For, if there were, then the mixed strategy profile equivalent to that profile would also be an equilibrium in mixed strategies.

Example 1

We first consider a game without perfect recall given in the next slide. (This is a very slight modification of a game considered by Kuhn (1953).) In this game there is a unique equilibrium in mixed strategies. For one of the players the equilibrium mixed strategy is not equivalent to any behaviour strategy.

Example 1: Extensive Form

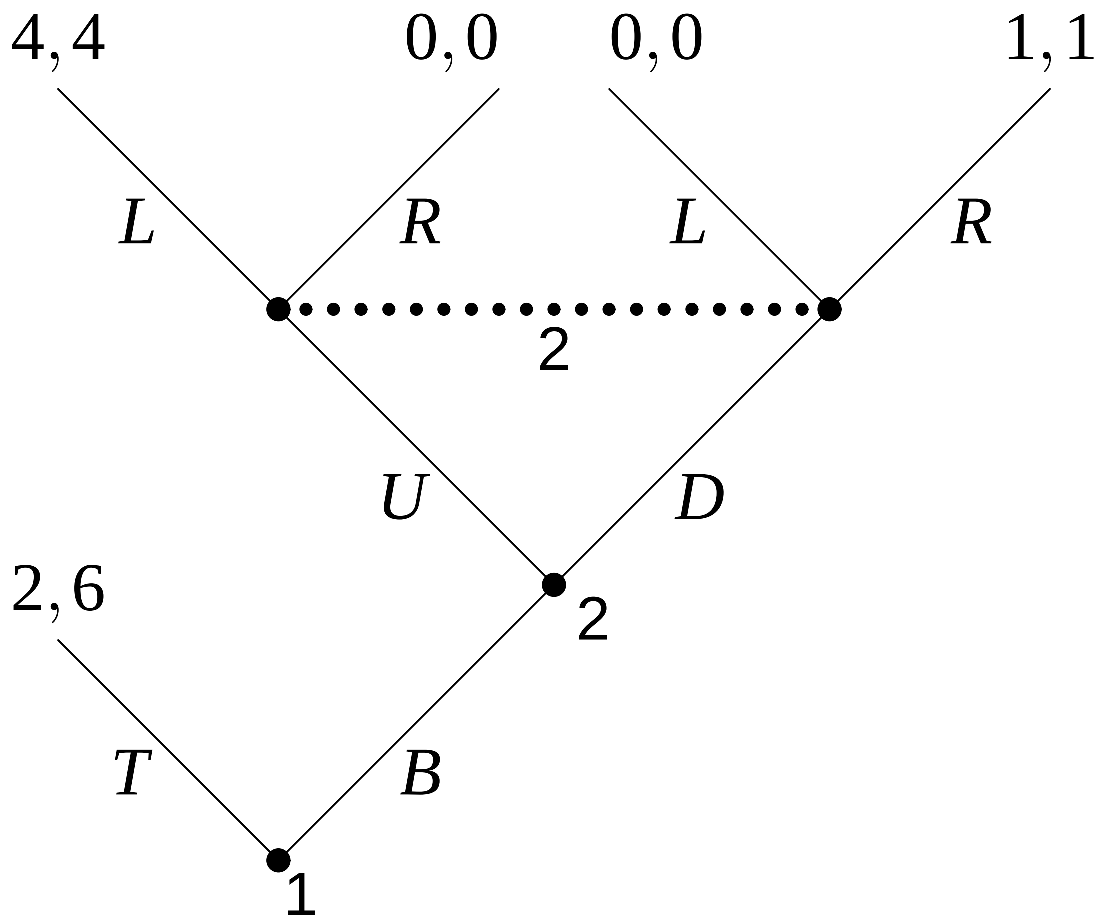


Example 1: Normal Form

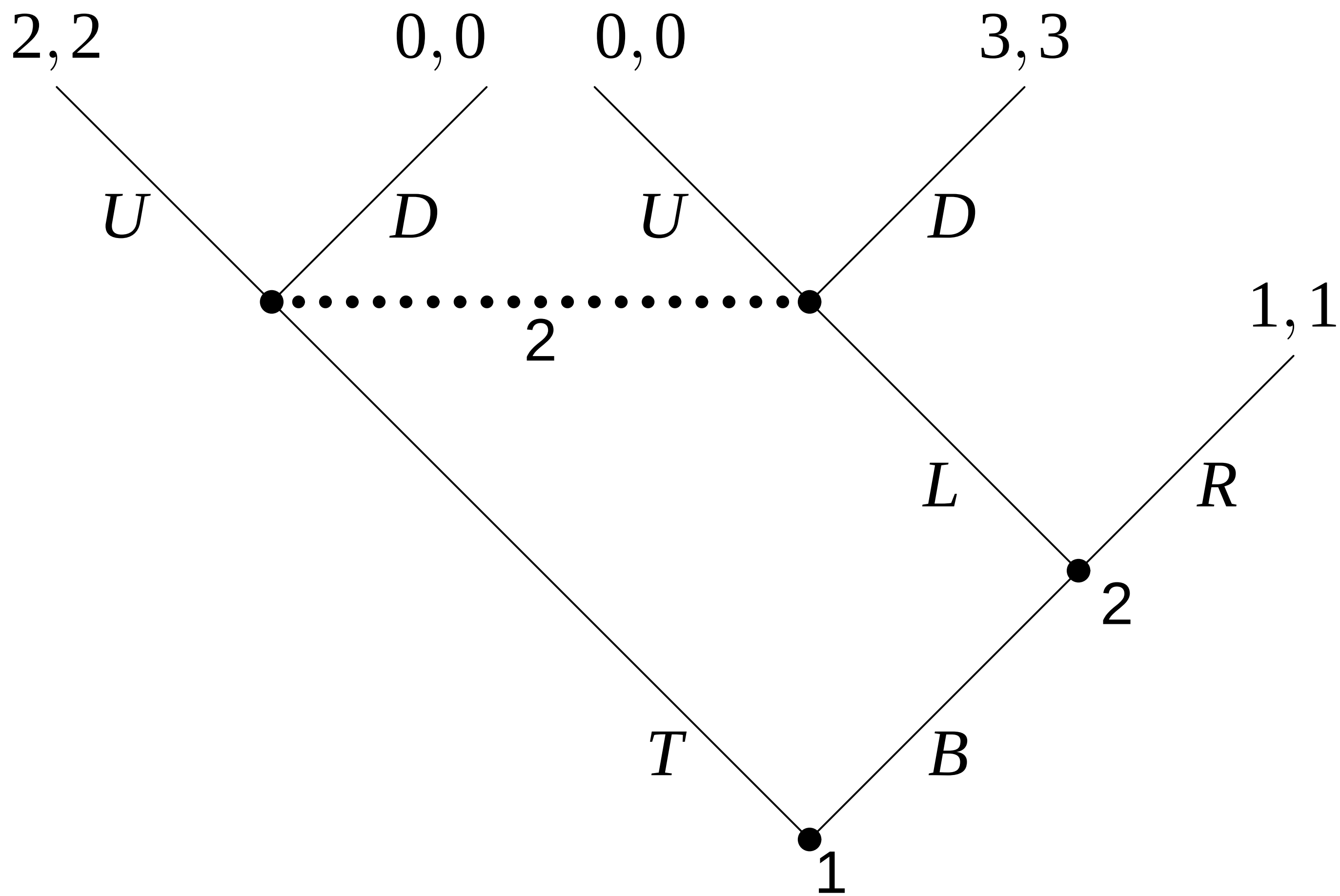
Player 2

Player 1		IN	OUT
	In, S	$0, 0$	$\frac{3}{2}, -\frac{3}{2}$
	In, D	$0, 0$	$-\frac{1}{2}, \frac{1}{2}$
	Out, S	$-\frac{3}{2}, \frac{3}{2}$	$0, 0$
	Out, D	$\frac{1}{2}, -\frac{1}{2}$	$0, 0$

Example 2: Extensive Form



Example 3



Cannot deviate at all information sets

Definitions of the Central Concepts

We shall now define sequential equilibria and quasi-perfect equilibria. Since we have seen that we cannot hope to satisfy a one-deviation property and that it will be necessary to consider players deviating simultaneously at a number of information sets we shall define beliefs not at an information set but at a collection of information sets. In the original definition of sequential equilibrium beliefs were defined as a probability distribution over the nodes of an information set. Here we define beliefs as distributions over the pure strategies that are being played, including Nature's "strategy." First we need a bit of notation.

Notation

We shall denote the set of players by N, the set of Player n's pure strategies by S_n, the set of Player n's behaviour strategies by B_n with $B = \times_{n \in N} B_n$, and the set of Player n's mixed strategies by Σ_n with $\Sigma = \times_{n \in N} \Sigma_n$. We shall let $S = \times_{n \in \{0\} \cup N} S_n$, that is, when we refer to profiles of pure strategies we shall specify also the "strategy" of Nature.

We shall denote the collection of information sets by $\mathcal{H}$, with $\mathcal{H}_n$ the information sets of Player n. We shall also consider the collection of non-empty subsets of $\mathcal{H}_n$ which we shall denote $\overline{\mathcal{H}}_n$. An element of $\overline{\mathcal{H}}_n$ is a collection of information sets of Player n.

System of Beliefs

Definition 5. A system of beliefs μ defines, for each n in N and each H in $\mathcal{H}_n$ a distribution $\mu(s_0, s_1, \ldots, s_N \mid H)$ over the profiles of pure strategies that reach H. Given μ we also consider $\mu_{S_n}(s_n \mid H)$ and $\mu_{S_{-n}}(s_{-n} \mid s_n, H)$ the marginal distribution on S_n given H and the conditional distribution on S_{-n} conditional on s_n and H.

Recall that we have seen above that a player's beliefs at an information set about what strategies the other players are playing may differ depending on what pure strategy he himself is playing. Notice also that we include the (pure) strategy of Nature in the list of strategies over which Player n has beliefs.

Sequential Equilibrium

We now define sequential equilibria.

Definition 6. Given a pair (σ, μ) we say that the pair is consistent (or is a consistent assessment) if there is a sequence of completely mixed strategy profiles $\sigma^t \to \sigma$ with μ^t a system of beliefs obtained from μ^t as conditional probabilities and $\mu^t \to \mu$.

Definition 7. Given a pair (σ, μ) we say that the pair is sequentially rational if, for each n, for each H in $\mathcal{H}_n$, and for each s_n in S_n if $\mu_{S_n}(s_n \mid H) > 0$ then s_n maximises

$$E_{\mu_{S_{-n}}(s_{-n}|s_n,H)} u_n(t_n, s_{-n})$$

over the set of all t_n in S_n such that t_n differs from s_n only at information sets in H.

Sequential Equilibrium

Definition 8. Given a pair (σ, μ) we say that the pair is a sequential equilibrium if it is both consistent and sequentially rational.

Quasi-perfect Equilibrium

Quasi-perfect equilibria are defined in a similar way.

Definition 9. A strategy profile σ is a quasi-perfect equilibrium if there is a sequence of completely mixed strategy profiles $\sigma^t \to \sigma$ with μ^t a system of beliefs obtained from μ^t as conditional probabilities and $\mu^t \to \mu$ and for each n, for each H in $\underline{\mathcal{H}}_n$, and for each s_n in S_n if $\mu_{S_n}(s_n \mid H) > 0$ then s_n maximises

$$E_{\mu^t_{S_{-n}}(s_{-n}|s_n,H)} u_n(t_n, s_{-n})$$

over the set of all t_n in S_n such that t_n differs from s_n only at information sets in H.

Observe that the definitions of sequential equilibrium and quasi-perfect equilibrium differ only in use of μ^t rather than μ in defining the expected utility that is maximised.

Results

The first two results say that in games with perfect recall we obtain the “same” equilibria as the original definitions.

Proposition 1. *If the game has perfect recall then if* (σ, μ) *is a sequential equilibrium then there is a behaviour strategy profile b, equivalent to* σ *in the sense of Kuhn, that is the strategy part of a sequential equilibrium according to the definition of Kreps and Wilson (1982). Moreover for any sequential equilibrium in the sense of Kreps and Wilson there is an equivalent mixed strategy* σ *and a system of beliefs* μ *such that* (σ, μ) *is a sequential equilibrium.*

Results

Proposition 2. *If the game has perfect recall then if* σ *is a quasi-perfect equilibrium then there is a behaviour strategy profile b, equivalent to* σ *in the sense of Kuhn, that is a quasi-perfect equilibrium according to the definition of van Damme (1983). Moreover, for any quasi-perfect equilibrium in the sense of van Damme there is an equivalent mixed strategy profile that is a quasi perfect equilibrium.*

More Results

The next two results say that the relation between the concepts is as it was in games with perfect recall.

Proposition 3. *Every quasi-perfect equilibrium is a sequential equilibrium.*

Proposition 4. *For a generic class of extensive form games every sequential equilibrium is a quasi-perfect equilibrium.*

Another Result

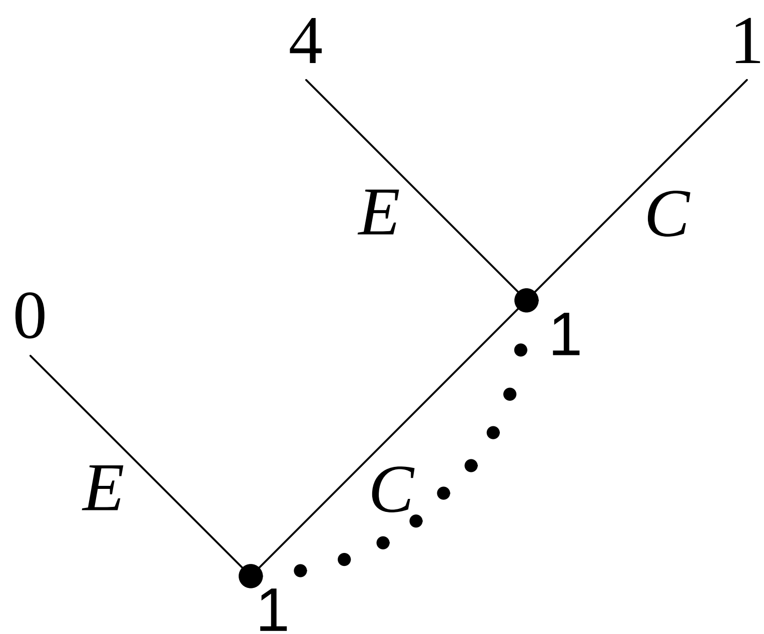
Finally we have the result proved for games with perfect recall by van Damme (1983) and Kohlberg and Mertens (1986) relating quasi-perfect and sequential equilibria to proper equilibria (Myerson 1978) of the normal form.

Proposition 5. *Every proper equilibrium is a quasi-perfect equilibrium (and hence a sequential equilibrium).*

Since every game has a proper equilibrium this result also implies the existence of sequential and quasi-perfect equilibria.

Nonlinear Games

We now turn to nonlinear games, that is, we remove the restriction that each [play of the game cuts each information set at most once.



The Absent Minded Driver

Nonlinear Games

Thus, in nonlinear games we need to use mixtures of behaviour strategies. Following Mertens, Sorin, and Zamir (2015) we call such mixtures general strategies.

Definition 10. A general strategy in an extensive form game for Player n is a probability distribution over the player's behaviour strategies.

In nonlinear games we also need to consider randomisations over behaviour strategies. We *can* consider such strategies for linear games, but we do not need to do so.

Definition 11. A general strategy in an extensive form game for Player n is a probability distribution over the player's behaviour strategies. We denote the set of Player n's general strategies by G_n and the set of general strategy profiles by $G = \times_{n \in N} G_n$.

Nonlinear Games

We have seen that in nonlinear games a player may achieve outcomes using behaviour strategies that he cannot achieve using pure or mixed strategies. Further he may need to conceal the particular behaviour strategy he is using from his opponents. Thus we are led to consider mixtures of behaviour strategies which we call general strategies. Since there are an infinite number of behaviour strategies the space of general strategies is infinite dimensional. Fortunately, we do not need to consider all mixtures of behaviour strategies. The following result allows us to restrict ourselves to a finite dimensional subset of G_n. This result was proved by Alpern (1988).

Nonlinear Games

Proposition 6 (Alpren). *For any Player n in N there is a finite finite number K_n such that for any general strategy g_n of Player n there is general strategy g'_n that puts weight on only on K_n elements of B_n that is equivalent to g_n in the sense that for any general strategies g_{-n} of the other players (g_n, g_{-n}) and (g'_n, g_{-n}) induce the same distribution on the terminal nodes.*

Nonlinear Games

Proof. Each terminal node t in T, the set of terminal nodes, defines a set of decision nodes of Player n on the path from the initial node to t. For each of these nodes there is a branch $\{x,y\}$ from x with y also on the path to t. A behaviour strategy b_n of Player n induces a conditional probability on the branch $\{x,y\}$ conditional on x having been reached. Let $q_n(t,b_n)$ be the product of the conditional probabilities generated by b_n on the branches following nodes owned by Player n that occur on the path to t. If Player n has no nodes on the path to t we let $q_n(t,b_n) = 0$. Similarly define $q_0(t,b_0)$ for Nature, where b_0 is Nature's only strategy. Thus if the players play $b = (b_1, b_2, \ldots, b_N)$the probability that terminal node t will be reached is $\prod_{n \in \hat{N}} q_n(t,b_n)$. Notice that b_n influences the distribution over terminal nodes only through $q_n(t,b_n)$.

Nonlinear Games

Let

$$Q_n = \{(q_t)_{t\in T} \subset [0,1]^T \mid \text{for some } g_n \text{ in } G_n \text{ for all } t$$
$$q_t = \int_{B_n} q_n(t,b_n)dg_n(b_n)\}.$$

It is clear that Q_n is the convex hull of those points $(q_t)_{t\in T}$ in $_n$ with g_n putting weight only on one behaviour strategy, that is, of the set

$$C_n = \{(q_t)_{t\in T} \subset [0,1]^T \mid \text{ for some } b_n \text{ in } B_n \text{ for all } t \ q_t = q_n(t,b_n)\}.$$

Nonlinear Games

But since C_n (and Q_n) are subsets of $\mathbb{R}^T$, by Carathéodory's Theorem any q in Q_n can be written is a convex combination of at most $T+1$ elements of C_n. That is it is generated by a general strategy g_n that puts weight on at most $T+1$ elements of B_n. We have shown that for any g_n we can find a g_n' that puts weight on only $T+1$ elements of B_n such that g_n and g_n' generate the same element of Q_n. But g_n will influence the probability of a final node only through the element of Q_n it generates and the result follows. $\square$

Remark 1. In our proof we have given $T+1$ as the bound on the number of behaviour strategies that may receive positive weight. This can be substantially strengthened. In general many different terminal nodes may be associated with the same set of edges following nodes of Player n on the path to that terminal node. We require in Q_n only one dimension for each such set of edges.

Remark 2. It is not true that we can restrict attention to only a fixed finite subset of B_n. Consider the Absent Minded Driver example we considered earlier. We claim that there is no finite set of behaviour strategies such that the outcome from any behaviour strategy can be replicated by some mixture over the given set. Suppose that we have T behaviour strategies $b^1, b^2, \ldots, b^T$ with $b^t = (x^t, 1-x^t)$ with x^t being the probability that the player chooses L. So, if the player plays b^t he ends up with outcome 0 with probability x^t, with outcome 4 with probability $x^t(1-x^t)$, and with outcome 1 with probability $(1-x^t)^2$. Let $\bar{x}$ be the smallest value of x^t strictly greater than 0.

Nonlinear Games

Consider the behaviour strategy $b^0 = (\bar{x}/2, 1 - (\bar{x}/2))$. This strategy gives outcome 0 with probability $\bar{x}/2$ and outcome 4 with probability $(\bar{x}/2)(1-(\bar{x}/2))$. Now for any b^t which give strictly positive probability of outcome 4 we have that the ratio of the probability of outcome 0 to the probability of outcome 4 is $1/(1-x^t) \geq 1/(1-\bar{x})$. Thus if we have a general strategy putting weight only on $b^1, b^2, \ldots, b^T$ that gives outcome 4 with the same probability as b^0, that is with probability $(\bar{x}/2)(1-(\bar{x}/2))$ it will give outcome 0 with probability at least

$$\left(\frac{1}{1-\bar{x}}\right)\left(\frac{\bar{x}}{2}\right)\left(1-\frac{\bar{x}}{2}\right) = \left(\frac{\bar{x}}{2}\right)\left(\frac{2-\bar{x}}{2-2\bar{x}}\right) > \frac{\bar{x}}{2},$$

and so it does not induce the same probabilities on outcomes as b^0.

Nonlinear Games

As a consequence of Proposition 6, instead of working with the infinite dimensional space G_n we can instead work with the finite dimensional space

$$\hat{G}_n = \Delta_{K_n} \times {B_n}^{K_n},$$

the Cartesian product of the K_n-simplex with K_n copies of B_n. The typical element $(\alpha_1, \ldots, \alpha_k, \ldots, \alpha_{K_n}, b_n^1, \ldots, b_n^k, \ldots, b_n^{K_n}) \in \hat{G}_n$ means that for each k Player n plays his behaviour strategy b_n^k with probability α_k. For every element of G_n there is a Kuhn-equivalent element in the subset $\hat{G}_n$. And again we let $\hat{G} = \times_{n \in N} \hat{G}_n$.

Nonlinear Games

We shall now define sequential equilibria and quasi-perfect equilibria. As in the case of linear games we cannot hope to satisfy a one-deviation property and that it will be necessary to consider players deviating simultaneously at a number of information sets. Again we shall define beliefs not at an information set but at a collection of information sets. However now we define beliefs as distributions over the behaviour strategies that are being played, including Nature's strategy, which we know.

Definition 12. A system of beliefs μ defines, for each n in N and each H in $\mathcal{H}_n$ a finite distribution $\mu(b_0, b_1, \ldots, b_N \mid H)$ over the profiles of behaviour strategies that reach H with strictly positive probability. Given μ we also consider $\mu_{B_n}(b_n \mid H)$ and $\mu_{B_{-n}}(b_{-n} \mid b_n, H)$ the marginal distribution on B_n given H and the conditional distribution on B_{-n} conditional on b_n and H.

Nonlinear Games

Recall that we have seen above that a player's beliefs at an information set about what strategies the other players are playing may differ depending on what behaviour strategy he himself is playing. Notice also that we include the "behaviour strategy" of Nature in the list of strategies over which Player n has beliefs.

We first define sequential equilibria.

Nonlinear Games

Definition 13. Given a pair (g, μ) we say that the pair is consistent (or is a consistent assessment) if there is a sequence of general strategies in $\hat{G}$ that put weight only on completely mixed behaviour strategies $g^t \to g$ with μ^t a system of beliefs obtained from g^t as conditional probabilities and $\mu^t \to \mu$.

Definition 14. Given a pair (g, μ) we say that the pair is sequentially rational if, for each n, for each H in $\mathcal{H}_n$, and for each b_n in B_n if $\mu_{B_n}(b_n \mid H) > 0$ then b_n maximises

$$E_{\mu_{B_{-n}}(b_{-n}|b_n,H)} u_n(\beta_n, b_{-n})$$

over the set of all β_n in B_n such that β_n differs from b_n only at information sets in H.

Nonlinear Games

Definition 15. Given a pair (g, μ) we say that the pair is a sequential equilibrium if it is both consistent and sequentially rational.

Nonlinear Games

Quasi-perfect equilibria are defined in a similar way.

Definition 16. A strategy profile g is a quasi-perfect equilibrium if there is a sequence of general strategies in $\hat{G}$ that put weight only on completely mixed behaviour strategies $g^t \to g$ with μ^t a system of beliefs obtained from g^t as conditional probabilities and $\mu^t \to \mu$. and for each n, for each H in $\mathcal{H}_n$, and for each b_n in B_n if $\mu_{B_n}(b_n \mid H) > 0$ then b_n maximises

$$E_{\mu^t_{B_{-n}}(b_{-n}|b_n,H)} u_n(\beta_n, b_{-n})$$

over the set of all β_n in B_n such that β_n differs from b_n only at information sets in H.

Observe that the definitions of sequential equilibrium and quasi-perfect equilibrium differ only in use of μ^t rather than μ in defining the expected utility that is maximised.

Nonlinear Games: Results

We now give a number of results about the concepts we have defined. The proofs are straightforward but are not yet included. The first two results say that in games with perfect recall we obtain the "same" equilibria as in linear games, and hence, in the light of earlier results, in games with perfect recall, the same as the solutions given by the original definitions.

Proposition 7. *If the game is a linear game then if (g, μ) is a sequential equilibrium then there is a mixed strategy profile σ, equivalent to g in the sense of Kuhn, that is the strategy part of a sequential equilibrium according to the definition given above for linear games. Moreover for any sequential equilibrium $\sigma, \mu)$ in that sense there is an equivalent general strategy g and a system of beliefs μ' such that (g, μ') is a sequential equilibrium in the sense defined in this part of the talk.*

Nonlinear Games: Results

Proposition 8. *If the game is a linear game then if g is a quasi-perfect equilibrium then there is a mixed strategy profile σ, equivalent to g in the sense of Kuhn, that is a quasi-perfect equilibrium according to the definition given above for linear games. Moreover for any sequential equilibrium $\sigma, \mu)$ in that sense there is an equivalent general strategy profile g and a system of beliefs μ' such that (g, μ') is a sequential equilibrium in the sense defined in this part of the talk.*
If the game has perfect recall then if σ is a quasi-perfect equilibrium then there is a behaviour strategy profile b, equivalent to σ in the sense of Kuhn, that is a quasi-perfect equilibrium according to the definition of van Damme (1984). Moreover, for any quasi-perfect equilibrium in that sense there is an equivalent general strategy profile g that is a quasi perfect equilibrium.

Nonlinear Games: Results

The next two results say that the relation between the concepts is as it was in games with perfect recall.

Proposition 9. *Every quasi-perfect equilibrium is a sequential equilibrium.*

Proposition 10. *For any extensive form, except for a semialgebraic set of payoffs of lower dimension than the set of all payoffs, every sequential equilibrium is a quasi-perfect equilibrium.*

Proof. The proof follows in a straightforward way similar to the proof of the generic equivalence of perfect and sequential equilibria in Blume and Zame (1994) and the proof, based on Blume and Zame, of the generic equivalence of quasi-perfect and sequential equilibria in Hillas, Kao, and Schiff (2016). □

Nonlinear Games: A Non-Result

In linear games we showed if a strategy in an extensive form game was proper in the associated normal form. There seems to be real difficulties in doing something similar for nonlinear games. The natural strategy to try is to treat behaviour strategies as pure strategies. However there is no way restrict to a particular finite set of behaviour strategies. In the absent minded driver game the particular behaviour strategy we need to use depends on the exact payoffs. Suppose we replace the terminal node at which Player 1 obtains 1 with a decision node of a second player who chooses between $(0, 0)$ and $(4, 0)$. Now the unique maximising behaviour strategy of Player 1 will trace out all behaviour strategies between playing E with probability 0 and playing E with probability $1/2$.

Nonlinear Games: A Non-Result

On the other adding behaviour strategies as new pure strategies will change the set of proper equilibria, including, in some games with perfect recall changing it away from the unique sequential equilibrium.

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