So far

Problem:

Given: bimatrix game (**A**,**B**). What are its Nash equilibria?

Overview:

Any equilibrium is a convex combination of extreme equilibria = certain vertices of polytopes derived from **A**, **B**.

Enumerate extreme equilibria (finitely many).

Output convex equilibrium components.

Best response polytope Q for player 2

	y_4	y_5
①	3	3
②	2	5
③	0	6

$= A$

$$Q = \{\, y \mid Ay \le 1,\ y \ge 0 \,\}$$

$$Q = \{\ (y_4, y_5)\ \mid$$

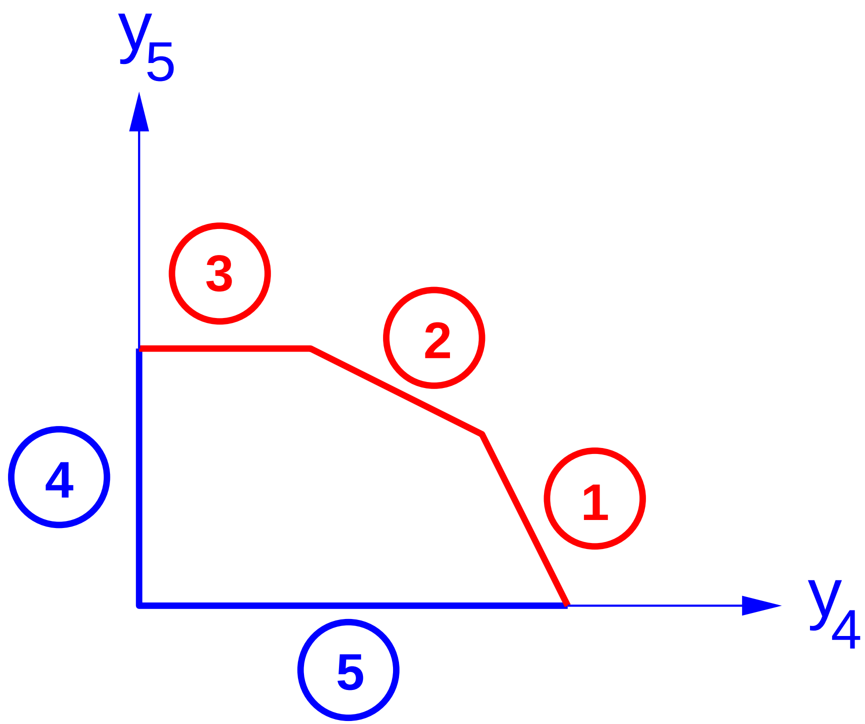
① : $3y_4 + 3y_5 \le 1$

② : $2y_4 + 5y_5 \le 1$

③ : $6y_5 \le 1$

④ : $y_4 \ge 0$

⑤ : $y_5 \ge 0\ \}$

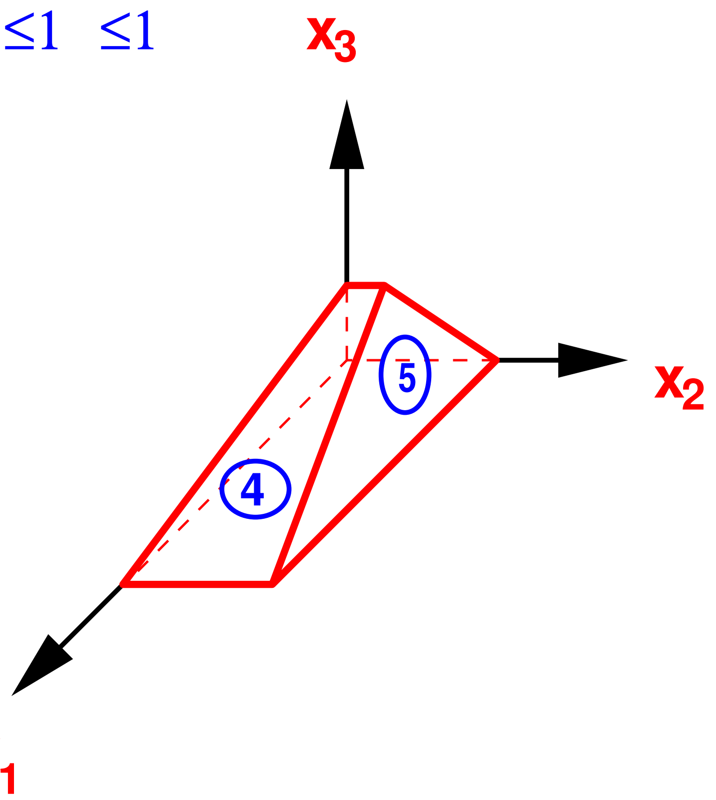


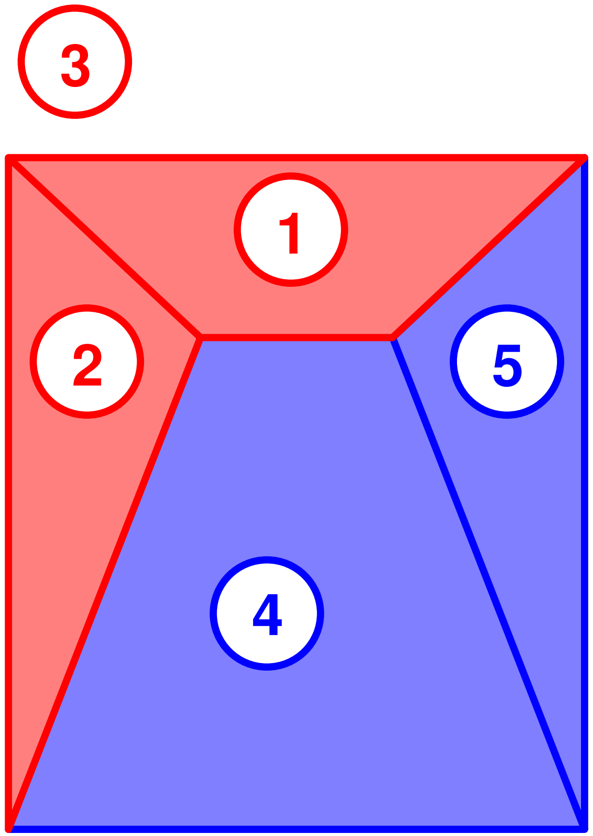
Best response polytope P for player 1

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 4 & 3 \end{bmatrix} = B$$

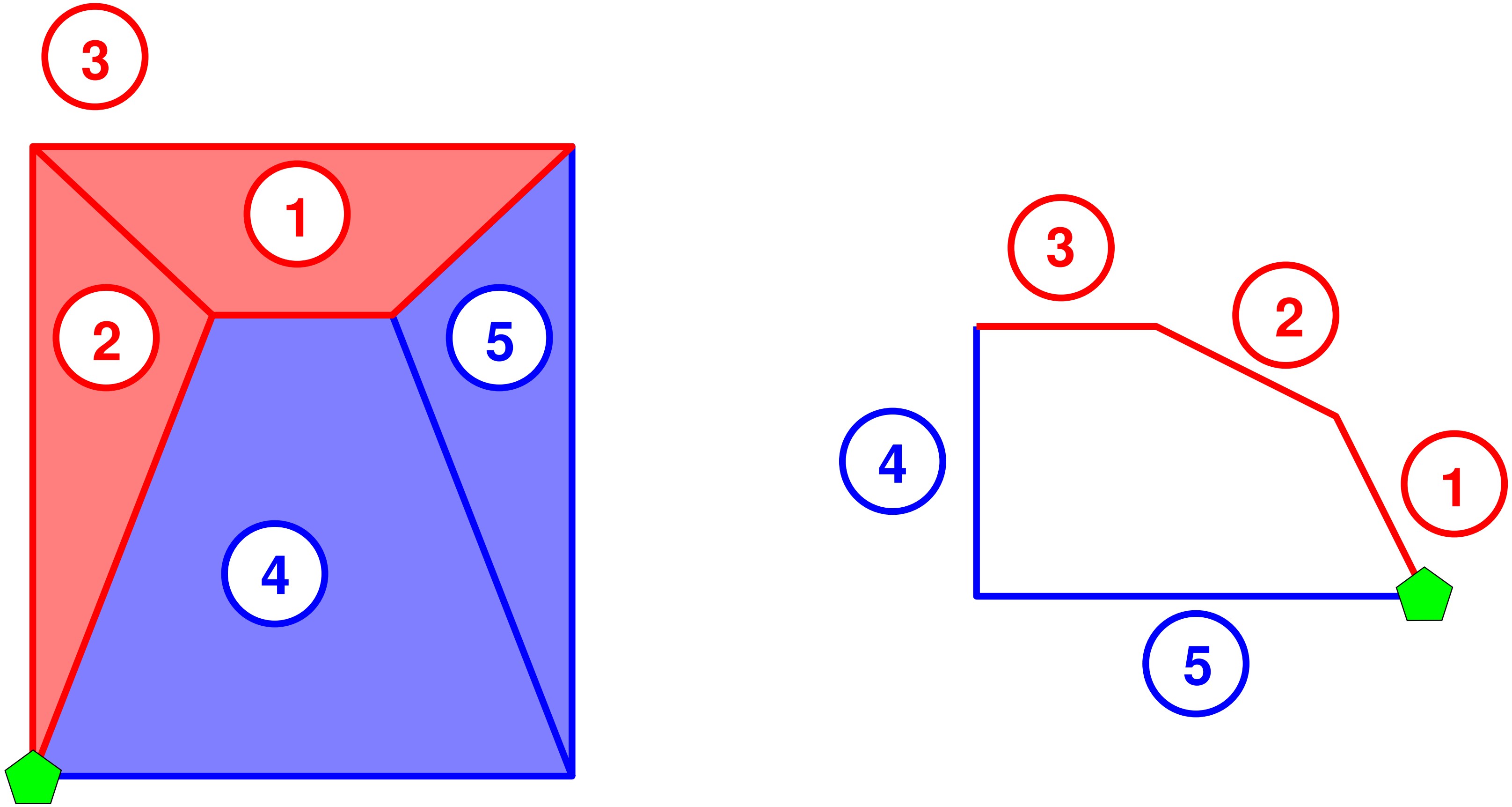
$$\begin{matrix} \le 1 & \le 1 \end{matrix}$$

$$P = \{\, x \mid x \ge 0,\ x^{\top} B \le 1 \,\}$$



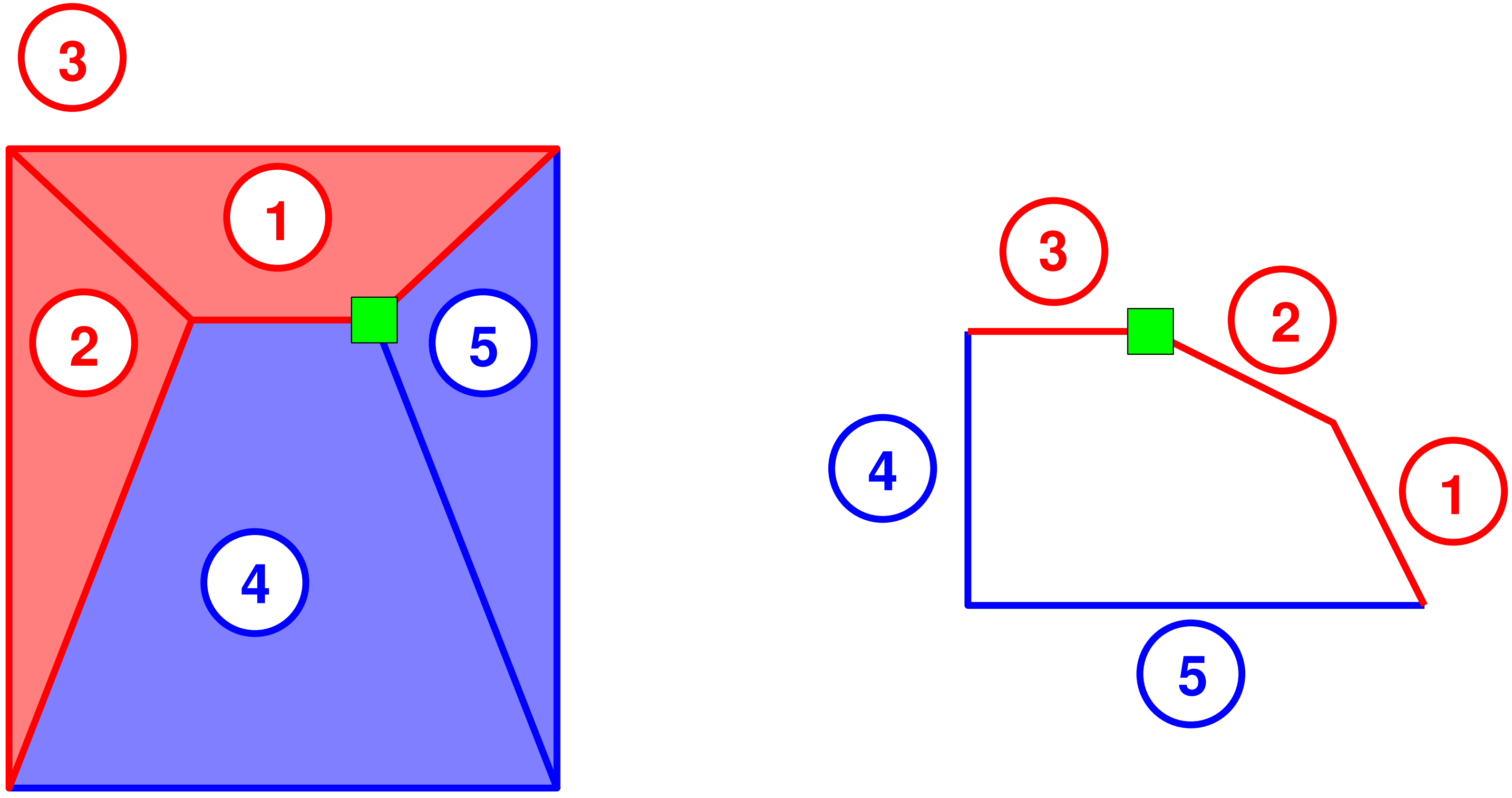


Equilibrium = completely labeled pair



pure equilibrium

Equilibrium = completely labeled pair



mixed equilibrium

Convex equilibrium components

[Winkels 1979 / Jansen 1980]

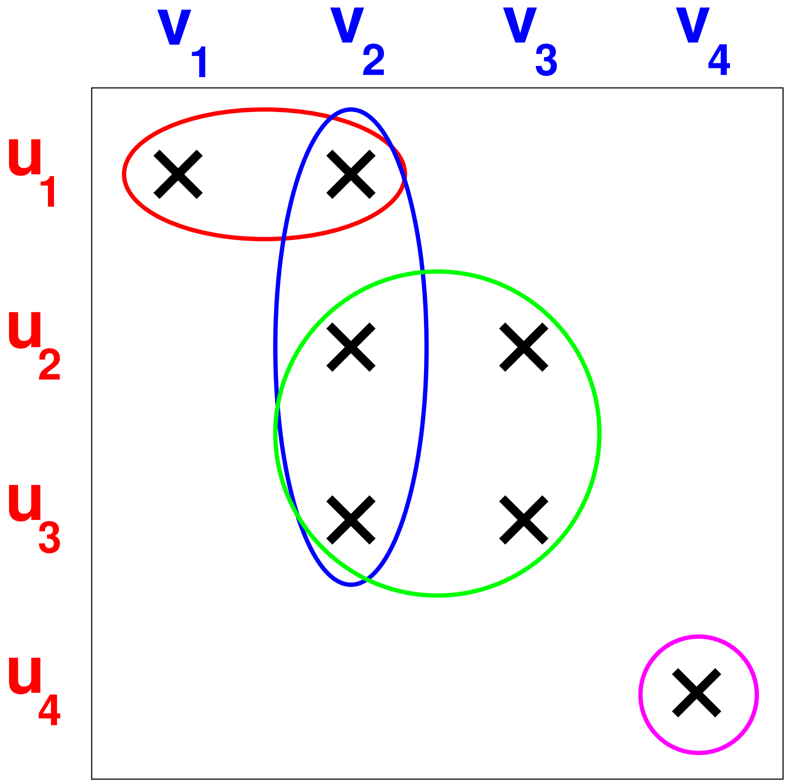
(**x**,**y**) is an equilibrium of (**A**,**B**) $\Leftrightarrow$

(**x**,**y**) is in the convex hull of **U** x **V** , where all (**u**, **v**) $\in$ **U** x **V** are completely labelled vertex pairs of **P** x **Q** **–** (**0,0**)

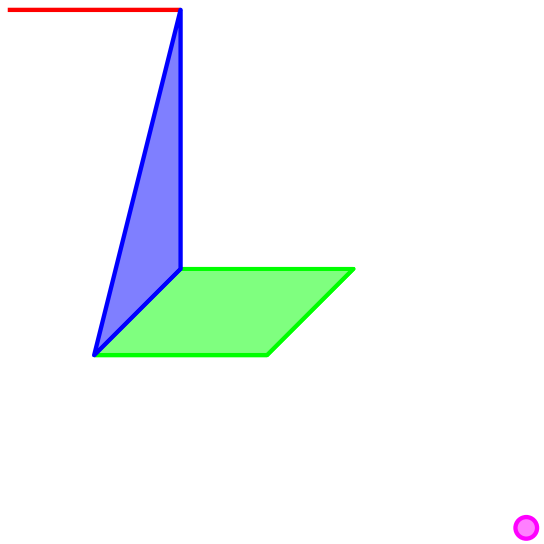
Convex equilibrium components U x **V**

= maximal cliques of bipartite graph

Convex equilibrium components



Geometry:



Clique enumeration

[Bron & Kerbosch 1973]

Recursive bottom-up generation of maximal cliques by elegant backtracking and branch and bound.

Adapted to bipartite graphs, outputs 2000 cliques / second independent of graph size.

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Simplex algorithm

solves a **linear program (LP)**:

maximize $\mathbf{c}\mathbf{x}$

subject to $\mathbf{x} \in \mathbf{P} = \{ \mathbf{x} \in \mathbf{R}^n \mid \mathbf{A}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \}$

- walks along edges of **P** from vertex to vertex by

  pivoting:

  equation dropped; edge traversed; new facet reached

- pivots chosen to improve objective function

Reverse search

[Avis & Fukuda 1994]

Input: **P** = { **x** $\in$ **R**^{n} **|** **Ax** $\leq$ **b** } (assume it's bounded)

Output: **vertex set** of **P**

Output efficient for simple (nondegenerate) polyhedra.

Reverse search

- Pick vertex **v** of **P** and **c** so that the LP: max **cx** s.t. **x** ∈ **P** is optimal for **x** = **v**.

- Simplex with unique pivoting rule (e.g., lexicographic rule [Dantzig, Orden, Wolfe 1959]) defines unique path from any vertex to **v**.

- Compute tree with root **v** by depth-first search, with reverse pivots (trial and error).

Reverse search