Problem Set 4 - Properties

1. By considering the trace of A^2, where A is the adjacency matrix of an (n, d, λ)-graph G, show that if $d \leq (1-\epsilon)n$ for some $\epsilon > 0$, then $\lambda \geq c(\epsilon)\sqrt{d}$ for some $c(\epsilon) > 0$ depending only on ϵ.

2. Show that the chromatic number of a (p, β)-jumbled graph is at least pn/β. Is this best possible?

3. Let G be a d-regular graph with n vertices and let $\lambda_1 \geq \cdots \geq \lambda_n$ be the eigenvalues of the adjacency matrix of A. Show that for every partition of the vertex set of G into two sets V_1 and V_2,
$$e(V_1, V_2) \leq \frac{dn}{4} - \frac{\lambda_n n}{4}.$$
[*Hint:* Let $x = (x_1, \ldots, x_n)$ be a vector with entries in ± 1 and consider $\sum_{(i,j) \in E(G)} (x_i - x_j)^2$.]

4. Show that if G is a (p, β)-jumbled graph on n vertices with $\beta = o(p^{t-1}n)$, then G contains $(1 + o(1))p^{\binom{t}{2}}\binom{n}{t}$ copies of K_t. How might this result be generalised to graphs other than K_t?