

Edge colouring multigraphs - Exercises 2

Problem 1:

Use Tashkinov's Theorem to prove Vizing's Theorem that $\chi'(G) \leq \Delta + \mu$ for every multigraph G .

Problem 2:

By following the suggested steps below, improve our proof of the "weak" version of Scheide's Theorem given in the lecture to the following statement: for every multigraph G we have

$$\chi'(G) \leq \max\{\Delta + \lceil \sqrt{(\Delta - 1)/2} \rceil, \lceil \rho(G) \rceil\}.$$

- (a) Show that if ϕ is a partial edge colouring of G and e is an uncoloured edge, then all maximal ϕ -Tashkinov trees starting with e have the same vertex set.
- (b) Show that there is a maximal Tashkinov tree T starting with e that uses at most $(|V(T)| - 1)/2$ colours on its edges.