

### Edge colouring multigraphs - Exercises 3

**Problem 1:**

Let  $G$  be a multigraph that is not bipartite. The *odd girth*  $g_0$  of  $G$  is the length of a shortest odd cycle in  $G$ . Prove the following generalisation of Shannon's Theorem:

$$\chi'(G) \leq \Delta + 1 + \frac{\Delta - 2}{g_0 - 1}.$$

(Hint: let  $e$  be an uncoloured edge with endpoints  $x$  and  $y$ , and suppose  $\alpha$  is a colour missing at  $x$  and  $\beta$  is missing at  $y$ . Let  $P$  denote the  $(\alpha, \beta)$ -alternating path starting at  $y$ . Then  $e$  together with all of  $P$  except the last vertex is a Tashkinov tree.)

**Problem 2:**

Suppose  $G$  is a multigraph such that  $\chi'(G) = \Delta + \mu$  where  $\mu \geq 2$ . Prove that  $G$  contains a triangle with vertices  $x, y$  and  $z$  such that  $xy$  has multiplicity  $\mu$  and  $xz$  has multiplicity at least  $\mu - 1$ .

(Hint: consider a maximal Kierstead path, that is, a Tashkinov tree that is a path.)