

Edge colouring multigraphs - Exercises 4

Problem 1:

In the Exercises for Lecture 3 we had the following problem:

Let G be a multigraph that is not bipartite. The *odd girth* g_0 of G is the length of a shortest odd cycle in G . Prove the following generalisation of Shannon's Theorem:

$$\chi'(G) \leq \Delta + 1 + \frac{\Delta - 2}{g_0 - 1}.$$

Find (and prove!) a complete characterisation of those connected multigraphs G for which

$$\chi'(G) = \Delta + 1 + \frac{\Delta - 2}{g_0 - 1}.$$

(In particular some divisibility conditions relating the parameters will be necessary.)