# Exercises: Day 1

**Exercise 1** (*) Let $\mathcal{P}, \mathcal{Q}$ be independent Poisson point processes in $[0,1]^2$ whose mean measures are $\lambda$ and $\nu$, respectively. Show that $\mathcal{P} \cup \mathcal{Q}$ is a Poisson point process in $[0,1]^2$ whose mean measure is $\lambda + \nu$.

**Exercise 2** (**) Let $\{\mathcal{P}_n\}_{n\in\mathbb{N}}$ be a sequence of Poisson point process in $[0,1]^2$ with $n$ times Lebesgue measure as their mean measure, and let $\mathcal{U}_n = \{u_1, \dots, u_n\}$ be $n$ points distributed uniformly at random in $[0,1]^2$.

- Show that for each $n$ and $m$, the law of $\mathcal{P}_n$ given that $|\mathcal{P}_n| = m$ is that of $\mathcal{U}_m$.
- Show that any event holding with probability at least $1 - o(n^{-1/2})$ in $\mathcal{P}_n$ holds with probability at least $1 - o(1)$ in $\mathcal{U}_n$.

**Exercise 3** (**) Let $\mathcal{P}_n$ be a Poisson point process with $n$ times Lebesgue measure as its mean measure in $[0,1]^2$, and let $r_n = \sqrt{\log n/\pi n}$. Let $\mathcal{G} = (\mathcal{P}_n, r_n)$ be the corresponding random geometric graph. Show that for every $\varepsilon > 0$, with high probability there are isolated vertices in $\mathcal{G} = (\mathcal{P}_n, (1-\varepsilon)r_n)$, and with high probability there are no isolated vertices in $\mathcal{G} = (\mathcal{P}_n, (1+\varepsilon)r_n)$.