Polarized and consensus fixed points in Boolean Networks

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December 11, 2023

Abstract

The process of opinion formation and influence on social networks is of great relevance as it is present in many aspects of life, such as politics, marketing, group decision-making, and more [1]. In particular, if we consider only binary opinions and deterministic interactions, this process can be modeled using Boolean networks [2] [3]. Boolean networks can be formally defined through a function $f : \{0,1\}^n \to \{0,1\}^n$ and are used in the modeling of complex systems.

We consider a system of agents with a fixed network, meaning that the connections and the type of influence are already determined. A stable state in this system is a set of opinions that does not change over time. If all agents have the same opinion, it is called a consensus stable state. If half of the agents have the opposite opinion to the other half, it's considered a polarized stable state. Finding these states is equivalent to finding the fixed points of the Boolean network that models the system [4], which is computationally hard [5] as the number of configurations increases exponentially with the number of agents.

The question that motivates the study is the possibility of characterizing consensus and polarized states of opinions in a society in the long term through the structure of the graph representing their interaction and the aggregation functions. To address this, two approaches to the problem are presented and related to typical graph problems that allow the analysis of their temporal complexity.

This work was carried out in collaboration with, **Julio Aracena**[1]

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*Partially funded by ANID, Beca Magister Nacional, e-mail: copazo2018@udec.cl
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