Load Balancing under Identical Concave Functions

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Abstract

In the field of combinatorial optimization algorithms, scheduling problems aim to assign jobs to machines. The load of a machine is the sum of the processing times of its assigned jobs. In this study, the goal is to analyze the complexity and develop algorithms for a load balancing problem, which consists of finding a job-to-machine assignment that maximizes the sum of an increasing concave function evaluated on the load.

The problem was first considered by Alon et al.[1], where an EPTAS (efficient polynomial time approximation scheme) is obtained for the *offline* version. This is essentially the best possible result since the problem is strongly NP-hard [2]. It is proven that in this context, the *Longest Processing Time* rule [3] achieves a $2(\sqrt{2}-1) \approx 0.828$ approximation. This analysis is not tight, providing only an upper bound of $11/12 \approx 0.916$. In the restricted case of having two machines, a tight analysis shows that this rule is exactly an $11/12$-approximation.

On the *online* version, jobs are revealed and assigned one by one. In adversarial order, it is proven that the greedy algorithm *List Scheduling* [3] is exactly $3/4$-competitive. It is also proven that this ratio is optimal among online algorithms that do not evaluate the concave function. Furthermore, a constant upper bound on the competitiveness of any online algorithm of $\phi/2 \approx 0.809$ is exhibited, with ϕ being the golden ratio. Finally, as a partial answer, an algorithm achieving this bound is proposed for the restricted case of two machines.

References

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