# A QPTAS for the Circle In-line Packing Problem

Elisa Dell'Arriva[1*] Flávio K. Miyazawa[1]

1 Institute of Computing, Universidade Estadual de Campinas, Brazil

**Abstract**

In the *circle in-line packing problem*, we have a list of circles and we must place them in a side-by-side manner over a horizontal line, such that: *i*) each circle touches the line only at its lowest point, and *ii*) there is no overlap between any two circles. The objective is to minimize the distance between the leftmost and rightmost points of the circles that define the extremes of the packing. See Figure 1 for an illustration.

This problem is NP-hard and we approached it via approximation algorithms. We say an algorithm $\mathcal{A}$ for a given (minimization) problem is an $\alpha$-*approximation*, where $\alpha$ is a positive constant, if the value of the solution given by $\mathcal{A}$ is at most $\alpha$ times the optimal value, for every instance of the problem. More strongly, in a *polynomial-time approximation scheme (PTAS)*, we have a constant $\varepsilon > 0$ and a polynomial-time algorithm $\mathcal{A}$ which is a $(1+\varepsilon)$-approximation. A QPTAS is the same of a PTAS, except that $\mathcal{A}$ is a quasi-polynomial-time algorithm.

In 2018, Dürr et al. [3] studied a version of the in-line packing problem where we have isosceles rectangle triangles instead of circles; they gave a QPTAS for the problem. Also in 2018, Alt et al. [1] gave a 4/3-approximation algorithm for the circle in-line packing problem and hinted that the QPTAS devired by Dürr et al. could possibly work for the version with circles as well. Dell'Arriva and Miyazawa [2] settled this question by presenting an adaptation of such QPTAS for the circle in-line packing problem. In this talk, we present such adaptation.

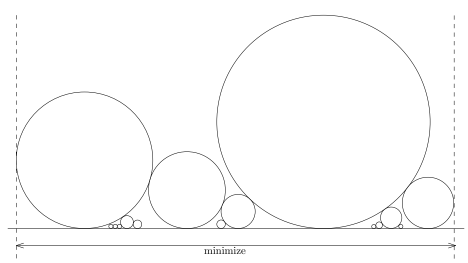


Figure 1: Illustration of an in-line packing of a set of circles.

## References

[1] Helmut Alt, Kevin Buchin, Steven Chaplick, Otfried Cheong, Philipp Kindermann, Christian Knauer, and Fabian Stehn. Placing your coins on a shelf. *Journal of Computational Geometry*, 9:312–327, 2018.

[2] Elisa Dell'Arriva and Flávio K. Miyazawa. Um qptas para o problema de empacotamento de círculos em linha. In *Anais do VIII Encontro de Teoria da Computação*, pages 114–118, Porto Alegre, RS, Brasil, 2023. SBC.

[3] Christoph Dürr, Zdeněk Hanzálek, Christian Konrad, Yasmina Seddik, René Sitters, Óscar C. Vásquez, and Gerhard Woeginger. The triangle scheduling problem. *Journal of Scheduling*, 21:305–312, 2018.