# $k$-Independence in Boolean Networks

**Julio Aracena**[*], **Raúl Astete**[†]
Departamento de Ingeniería Matemática
Universidad de Concepción

**Abstract**

In combinatorial testing, binary covering arrays of strength $k$ are introduced as a tool to test algorithms that produce incorrect results due to the interaction of $k$ parameters. More specifically, a matrix $A$ with $m$ rows and $n$ columns, with elements in $\{0,1\}$, is called a covering array of strength $k$ if, for any subset of $k$ columns, in the rows indexed by that selection, all vectors from $\{0,1\}^k$ appear. On the other hand, a Boolean network is a system of $n$ variables that interact with each other and evolve discretely over time according to a predefined rule. Formally, a Boolean network (BN) is a function $f : \{0,1\}^n \to \{0,1\}^n$, where $f(x) = (f_1(x), \ldots, f_n(x))$ for $x \in \{0,1\}^n$, and these functions $f_i : \{0,1\}^n \to \{0,1\}$ correspond to local activation functions. Additionally, we define the interaction digraph of the BN as $G = (V, A)$, with $V = \{1, 2, \ldots, n\}$ so that the arc $(i, j)$ exists when the function $f_j$ depends on $x_i$. An intriguing focus lies in studying configurations $x \in \{0,1\}^n$ where $f(x) = x$, signifying fixed points of the network.

In diverse applications, particularly in genetic networks, Boolean networks play a crucial role as fixed points resemble cellular phenotypic configurations. Research has predominantly explored the correlation between the number of steady states and properties of local activation functions. However, limited exploration exists regarding the insights into the architecture of a Boolean network derived from structural properties of its fixed points. In this context, our research aims to deepen the understanding of qualitative properties of fixed points, focusing on Boolean networks whose stable states constitute the rows of a covering array with specific strength, and exploring the implications of this property on the interaction graph.

[*]Parcialmente financiado por Proyecto BASAL.-ANID PFB 210005 “Center for Mathematical Modeling”, Universidad de Chile, e-mail: `jaracena@udec.cl`
Departmento de Ingeniería Matemática
Universidad de Concepción
[†]Parcialmente financiado por ANID, Beca Magíster Nacional, e-mail: `rastete2018@udec.cl`