Titles and Abstracts Dyadisc 7
(Chile program)
December 9 to 13, 2024

December 2, 2024

1 Common Talks

- **Julien Boulanger:**

  Title: Hecke continued fractions and connection points on Veech surfaces

  Abstract: The purpose of the talk is to introduce so called Hecke continued fractions and explain a correspondence between real numbers having finite (resp. eventually periodic) Hecke continued fraction expansion and slopes of vertex-to-vertex trajectories on double regular polygon translation surfaces (following Veech, and Schmidt-Sheingorn). We will use this correspondence to obtain results on so-called "connection points" on translation surfaces, and finally give an application to billards trajectories in regular polygons as well as a few conjectures (if we have time).

- **Irene Inoquio: Title:** Statistical Properties of Expanding Circle Maps with an Indifferent Fixed Point

  Abstract: In the setting of expanding maps of the circle with an indifferent fixed point, we study the joint behavior of the dynamics and pairs of moduli of continuity (ω, Ω), which ω is related to the regularity classes of potentials and Ω the regularity of observables. This interaction plays a central role in the development of equilibrium theory. In this talk, we remark on a specific property of the modulus Ω that guarantees favorable statistical properties for the associated equilibrium states, such as exponential decay of correlations and the Central Limit Theorem. Additionally, we present some preliminary elements that suggest the formulation of a Large Deviation Principle for equilibrium states linked to moduli of continuity. This is joint work with E. Garibaldi.

- **Cristobal Rivas:**

  Title: Entropy for one-dimensional group actions

Abstract: In this talk I will discuss the notion of entropy in the one dimensional setting. It is well known, that for a single homeomorphism entropy is always zero, yet this is not necessarily the case for more general group actions. In particular, entropy is positive as soon as the group contains "crossed elements", and it is a result from Ghys, Langevin and Walczak, that in the C^2 setting crossed elements is the only mechanism for producing entropy. This is no longer true in the C^0 setting, and it is an open problem in the C^1 setting. My aim for the talk is to explain the notions above, and describe some mechanisms for producing entropy in the absence of crossed elements.

This is a ongoing project with Michele Triestino, Andrés Navas and Nicolas Matte Bon.

2 Course in Chile

- **Mélodie Andrieu:**

  Title: Infinite words of very low complexity

  Abstract: This lecture is an introduction to combinatorics on words and its interactions with dynamics, algebra, and arithmetic. The chosen perspective is that of complexity. By definition, the complexity of an infinite word is the function that counts, for every integer n, the number of subwords of length n found in the word w. For instance, the complexity of the infinite word $w = 0000000000\ldots$ is the constant function equal to 1, since for each integer n, w has a unique subword of length n, namely, the sequence of n zeros. Clearly, this complexity function is the lowest possible for an infinite word. The questions that will guide our investigation are: What is the minimum complexity for a "non-trivial" binary, ternary, or higher-ary infinite word? (And, first, what does "non-trivial" mean?) What are the words that achieve such complexity? Are there many? Are they interesting? In exploring these questions, we will introduce classic objects and tools from combinatorics on words (e.g., Sturmian words and Rauzy graphs), as well as strong, lesser-known results (notably a theorem by Tijdeman from 1999, which generalizes a seminal result of Morse and Hedlund from 1938, and for which we will provide an alternative proof). We will also present long-standing conjectures, and more recent open questions.

3 Talks in Chile

- **Álvaro Bustos**

  Title: Factors of substitutive shift spaces and the column semigroup

  Abstract: Substitutive shift spaces are a large and well-known class of dynamical systems of a self-similar nature, providing tractable examples

of interesting dynamical behaviour combined with very low complexity (and thus zero entropy). In this talk, we discuss the subclass of shift spaces originating from primitive constant-length substitutions; systems of this kind have an associated algebraic object, the column semigroup, which despite not being a conjugacy invariant dictates several aspects of the dynamics of the system (e.g. its automorphism group, the cardinality of the fibres of its maximal equicontinuous factor, etc.). Using this semigroup, we give an algebraic description of some dynamical properties of a constant-length substitutive shift space, with a focus on the existence of substitutive factors with specific properties. We discuss necessary and sufficient conditions for the existence of either a bijective factor or a Toeplitz factor, and show how they both can be described in terms of the ideal structure of the column semigroup, and how this leads to an algorithm to compute such a factor, if one exists, and decide whether this is not the case. This is a joint work with Johannes Kellendonk and Reem Yassawi.

- **Christopher Cabezas:**

  Title: Decidability of the isomorphism problem between constant-shape substitutions

  Abstract: An important question in dynamical systems is the classification, i.e., to be able to distinguish two isomorphic dynamical systems. In this work, we focus on the family of multidimensional substitutive subshifts. Constant-shape substitutions are a multidimensional generalization of constant-length substitutions, where any letter is assigned a pattern with the same shape. We prove that in this class of substitutive subshifts, under the hypothesis of having the same structure, it is decidable whether there exists a factor map between two aperiodic minimal substitutive subshifts. The strategy followed in this work consists in giving a complete description of the factor maps between these substitutive subshifts. We will also discuss related results, such as a condition to ensure that the substitution defines a subshift, and some consequences on coalescence, automorphism group and number of symbolic factors. This is a joint work with Julien Leroy.

- **Haritha Cheriyath**

  Title: Asymptoticity, automorphism groups and orbit equivalence

  Abstract: Two topological dynamical systems are said to be orbit equivalent if there exists a homeomorphism from one state space to another that induces a bijective correspondence between their orbits. Building on the celebrated result of Giordano, Putnam, and Skau, which characterizes the orbit (or strong orbit) equivalence of minimal Cantor systems using dimension groups, several works have been done on characterizing the dynamical invariants in this context. This talk aims to introduce the study of asymptotic components and automorphism groups within strong orbit equivalence (SOE) classes. We illustrate that within any given SOE

class of minimal Cantor systems, one can find subshifts with asymptotic components of various cardinalities - any finite number, countably infinite, or uncountable. Consequently, any SOE class contains a subshift of zero entropy whose automorphism group is isomorphic to Z. This talk is based on joint work with Sebastián Donoso.

- **María Isabel Cortez:**

  Title: Furstenberg-Weiss type almost 1-1 extensions: Toeplitz subshifts as a test family for amenability

  Abstract: In 1969, Jacobs and Keane introduced Toeplitz subshifts in the context of Z actions. Since then, different works have shown that there are dynamical systems within this family of subshifts with varied behavior. For example, Downarowicz has shown in 1991 that any possible set of invariant probability measures is realizable by some Toeplitz subshift in $\{0,1\}^Z$. The flexibility of Toeplitz subshifts motivated further generalizations beyond Z-actions, providing examples of group actions on the Cantor set with exciting properties. Another consequence of the generalization of the Toeplitz subshift concept is the characterization of infinite residually finite groups as those admitting actions corresponding to non-periodic Toeplitz subshifts. In this talk, applying results concerning Furstenberg-Weiss type almost 1-1 extensions, we show that Toeplitz subshifts are a test family for the amenability of residually finite groups (joint work with Jaime Gómez).

- **Bastián Espinoza:**

  Title: Dynamics of dendric subshifts

  Abstract: Dendric subshifts are a recently introduced generalization of both interval exchange transformations and Arnoux-Rauzy subshifts that preserve some of their main properties, and thus represent an exciting class of objects from dynamical, geometric, and combinatorial perspectives. In this talk, I will present new results concerning dynamical factors and S-adic structure of dendric subshifts.

- **Anahí Gajardo:**

  Title: Low complexity subshifts associated to one dimensional Turing machines

  Abstract: When studying the topological dynamics of Turing machines, a key tool is to look at the subshift obtained by looking the sequence of states and symbols that the head attains, this subshift contains all the necessary information about the system. In this subsgi, the entropy is only given by the configurations where the machine has positive velocity, if these configurations have low complexity, then the subshift has 0 entropy. Within the class of machines with null entropy are the subtitutive machines and the aperiodic machines. In this talk we will what we know about these machines and their subshifts.

- **Till Hauser:**

  Title: Mean equicontinuous factor maps

  Abstract: Mean equicontinity is a well studied notion for actions. We propose a definition of mean equicontinuous factor maps that generalizes mean equicontinuity to the relative context. For this we work in the context of countable amenable groups. We show that a factor map is equicontinuous, if and only if it is mean equicontinuous and distal. Furthermore, we show that a factor map is topo-isomorphic, if and only if it is mean equicontinuous and proximal. We present that the notions of topo-isomorphy and Banach proximality coincide for all factor maps. In the second part of the paper we turn our attention to decomposition and composition properties. It is well known that a mean equicontinuous action is a topo-isomorphic extension of an equicontinuous action. In the context of minimal and the context of weakly mean equicontinuous actions, respectively, we show that any mean equicontinuous factor map can be decomposed into an equicontinuous factor map after a topo-isomorphic factor map. Furthermore, for factor maps between weakly mean equicontinuous actions we show that a factor map is mean equicontinuous, if and only if it is the composition of an equicontinuous factor map after a topo-isomorphic factor map. We will see that this decomposition is always unique up to conjugacy.

- **Felipe Hernández:**

  Title: Multiple Polynomial Recurrence in Weyl Systems.

  Abstract: In this on-going work we investigate polynomial multiple recurrence, with a particular focus on Weyl systems, providing a comprehensive characterization of this phenomenon in such family of systems. Our analysis yields examples of averaging sets of multiple recurrence that are good for specific families of polynomials but not to others. Furthermore, we establish that, in Weyl systems, measurable recurrence and topological recurrence coincide. These results are obtained through an in-depth study of Weyl complexity and the introduction of the new concept of a Weyl space generated by a family of polynomials.

- **Tristán Radic**

  Title: Partial rigidity rate in constant-length s-adic subshifts

  Abstract: For a dynamical system $(X, \mathcal{X}, \mu, T)$ the partial rigidity rate is the greatest constant $\delta > 0$ for which there is a sequence $(n_k)_{k\in N}$ such that $\liminf_{k\to\infty} \mu(A \cap T^{n_k} A) \geq \delta\mu(A)$ for every $A \in \mathcal{X}$. The partial rigidity rate is an isomorphism invariant and provides a quantitative notion of recurrence. In this talk, we first explore methods to compute the partial rigidity rate for constant-length $\mathcal{S}$-adic subshifts. We then construct minimal and finite rank systems with distinct ergodic measures each one with distinct partial rigidity rates. This talk is based on two papers, the first of which is joint work with Alejandro Maass and Sebastián Donoso.

- **Víctor Sirvent:**

  Title: Generalised Fibonacci shifts in the Lorenz attractor

  Abstract: In this talk we deal with symmetric Lorenz attractors having a homoclinic loop that exhibits a well ordered orbit. We show the symmetry implies a very regular behaviour on the dynamic in the topological and metric sense. Let $([-1,1],f)$ be the one-dimensional reduction Lorenz map satisfying a well ordered orbit and $([-1,0],\tilde{f})$ be the quotient map, given by the equivalence relation $x \sim -x$, the dynamic of $\widetilde{f}$ is described explicitly as a subshift of finite type which generalizes the Fibonacci shifts and this fact is used to compute topological entropy of f.

  Moreover we show that in general $([-1,0],\tilde{f})$ is related to a factor of the k-bonacci shift. In particular we found that the 1-dimensional Lorenz map replicates an interesting duplicating behaviour of the k-bonacci shift found in different contexts.

- **Vicente Saavedra:**

  Title: Distribution of integers with digit restrictions in arithmetic progressions

  Abstract: For a given base $g \geq 2$ and a set of digits $\mathcal{D} \subset \{0,\ldots,g-1\}$, we denote by $\mathcal{C}_{g,\mathcal{D}}$ the set of positive integers that can be written in base g only using digits from $\mathcal{D}$. This class has been widely studied, including its distribution in arithmetic progressions. In this talk, we will discuss some classic results in the area and provide a necessary and sufficient condition for $\mathcal{C}_{g,\mathcal{D}}$ to be uniformly distributed in congruence classes. Furthermore, we will extend this analysis to more general digit restrictions through an identification with the language of sofic subshifts.