Transversal to horocycle flow on the moduli space of doubled slit tori

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Expanding Dynamics

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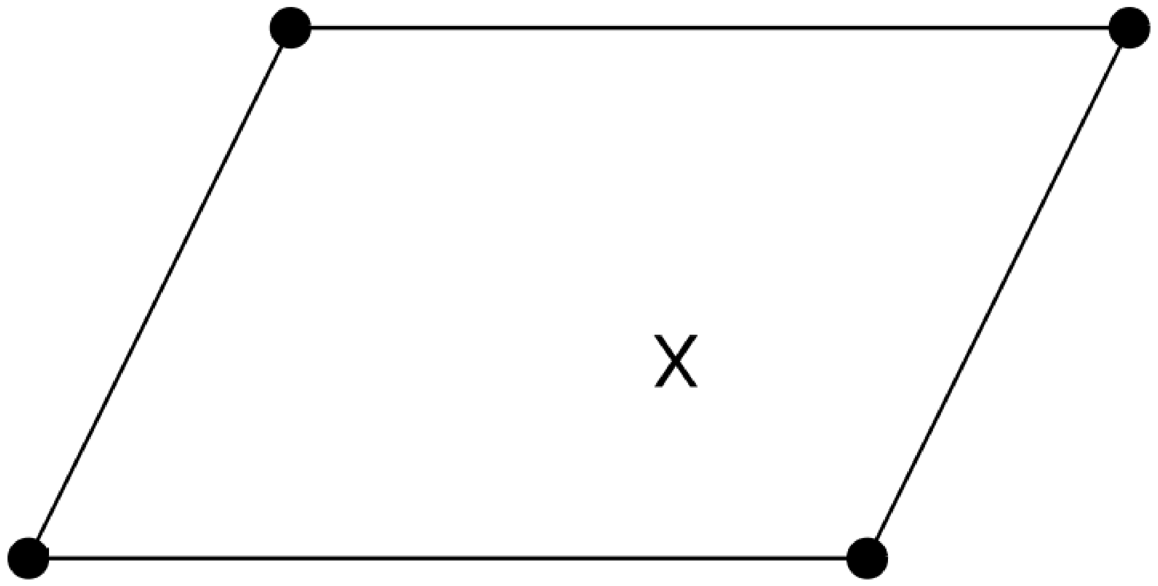
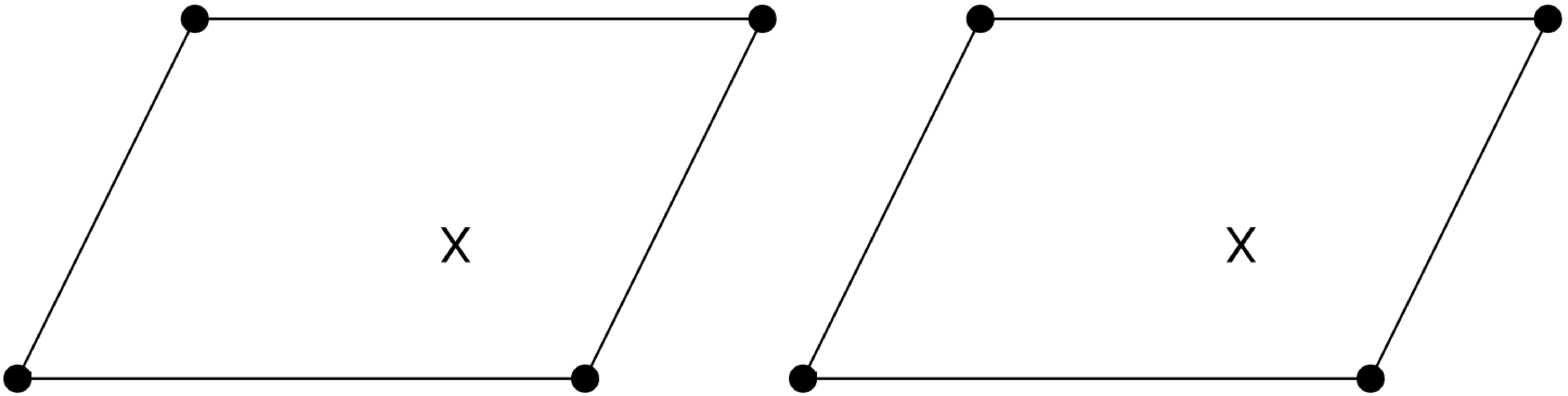
Doubled slit torus construction

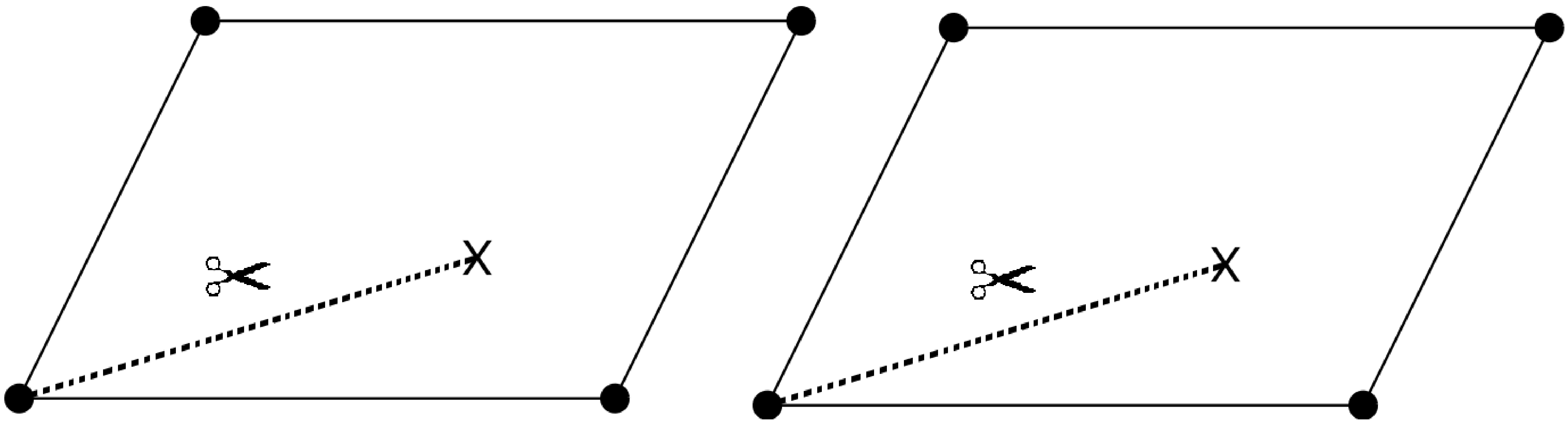
Take a flat torus and mark two points



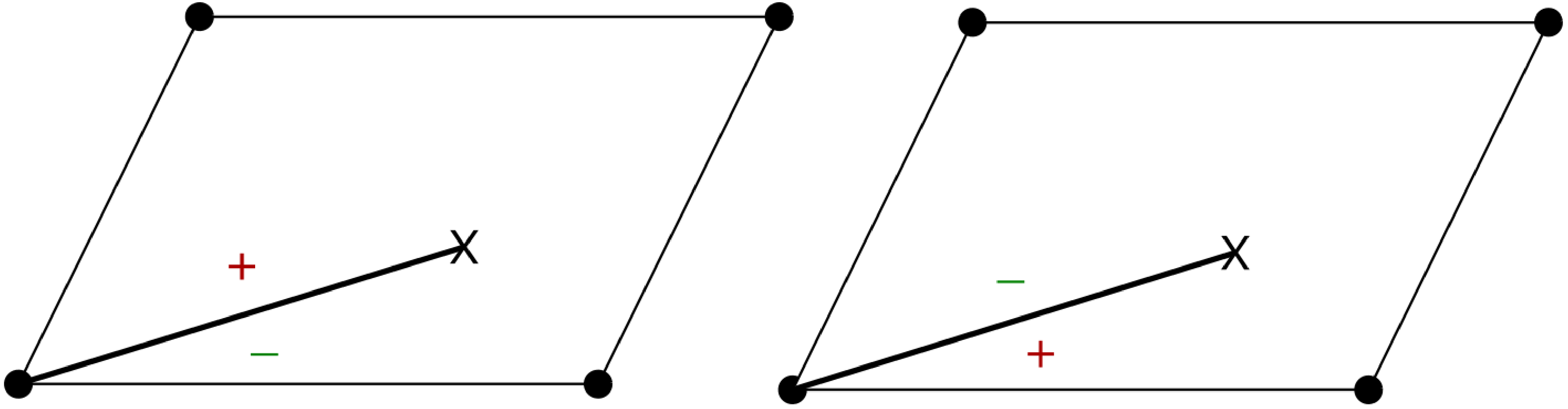
Take an identical copy of the twice-marked torus



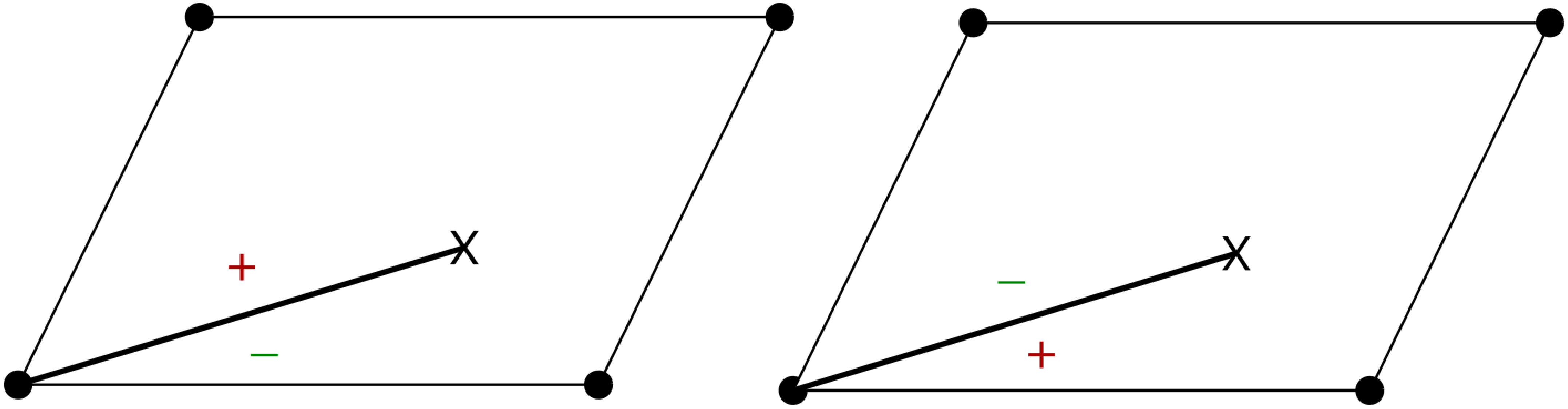
Cut a slit between the marked points



Glue opposite sides of the slit together



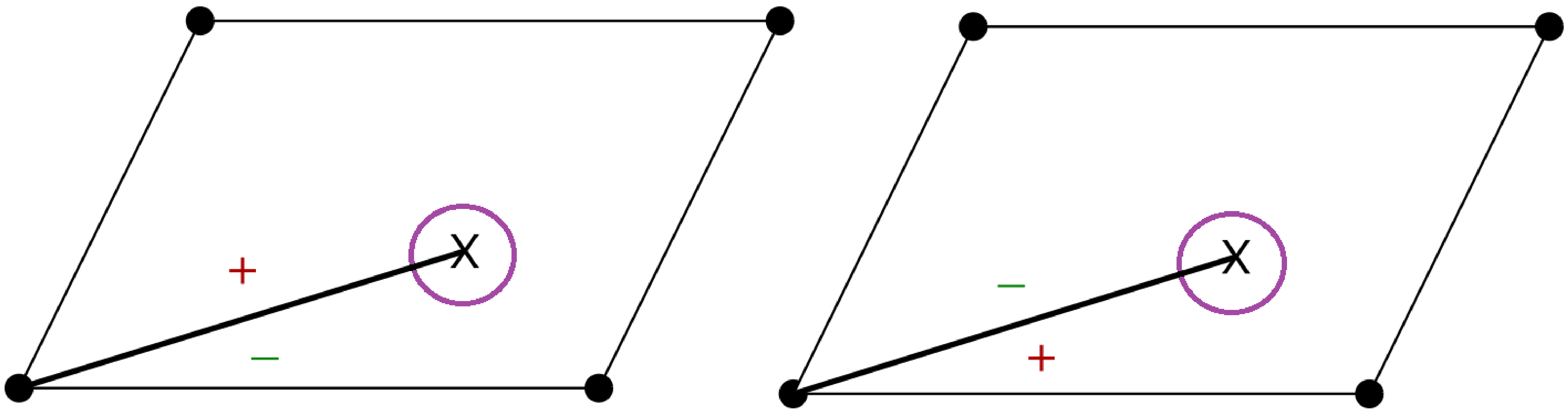
Doubled Slit Torus



Genus 2 surface

2 cone type singularities of angle 4π

Doubled Slit Torus



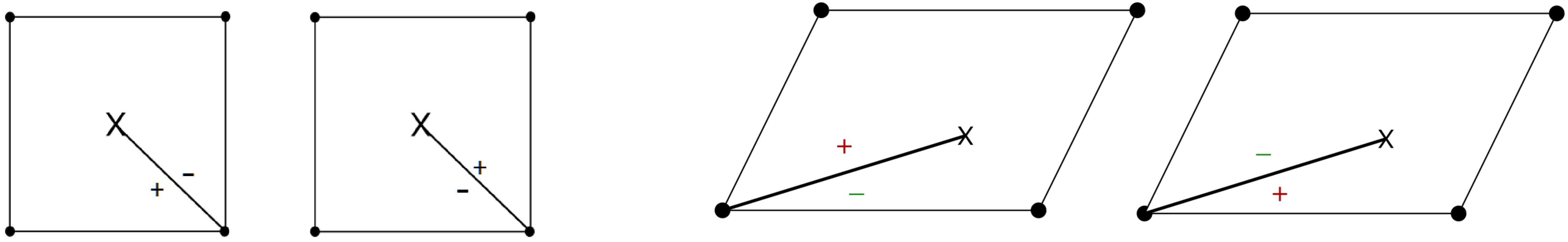
Genus 2 surface

2 cone type singularities of angle 4π

Transversal to horocycle flow on the moduli space of doubled slit tori

The moduli space $\mathcal{E}$

Let $\mathcal{E}$ denote the set of all doubled slit tori



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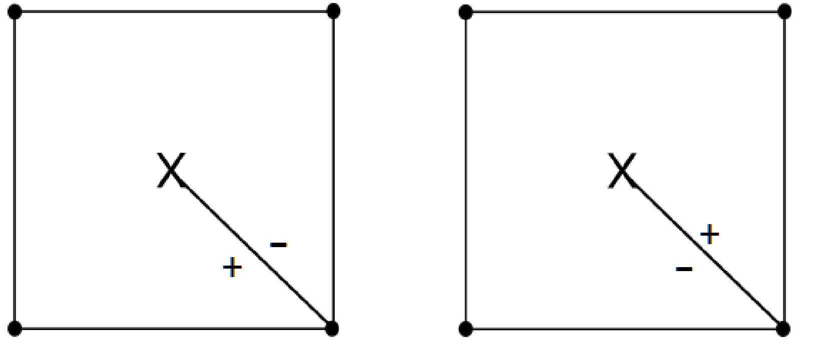
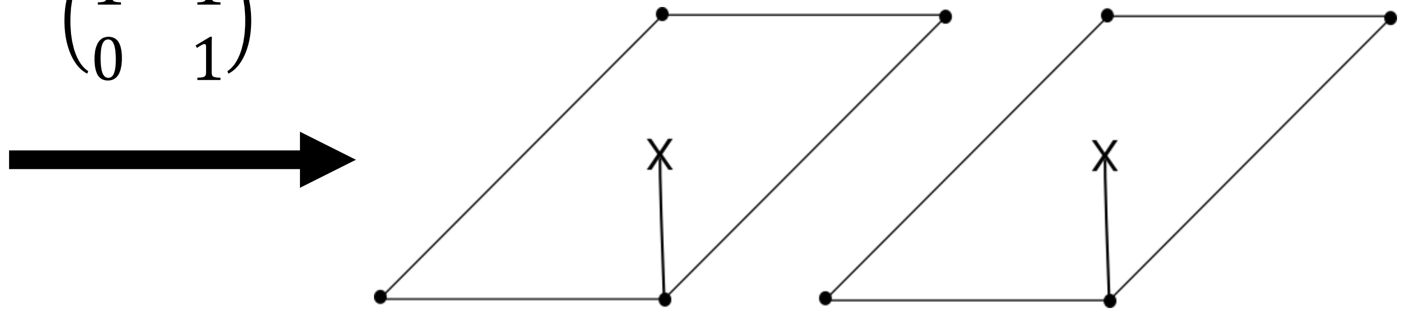
The $SL(2,\mathbb{R})$-action

There is a “linear” action of $SL(2,\mathbb{R})$ on $\mathcal{E}$:

act on the polygon



$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

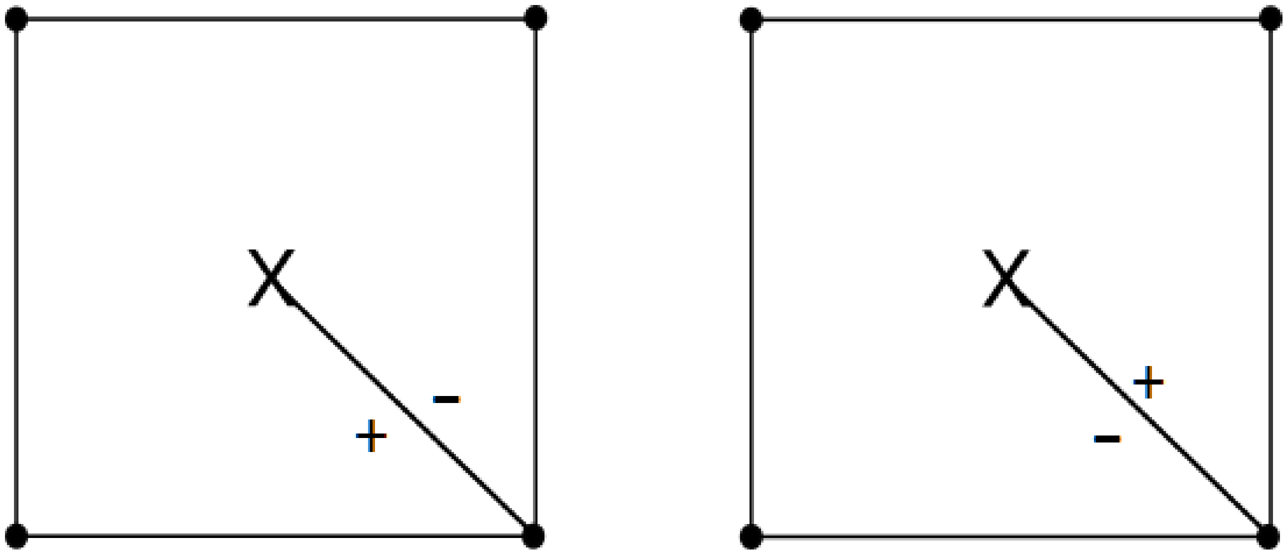


Horocycle flow

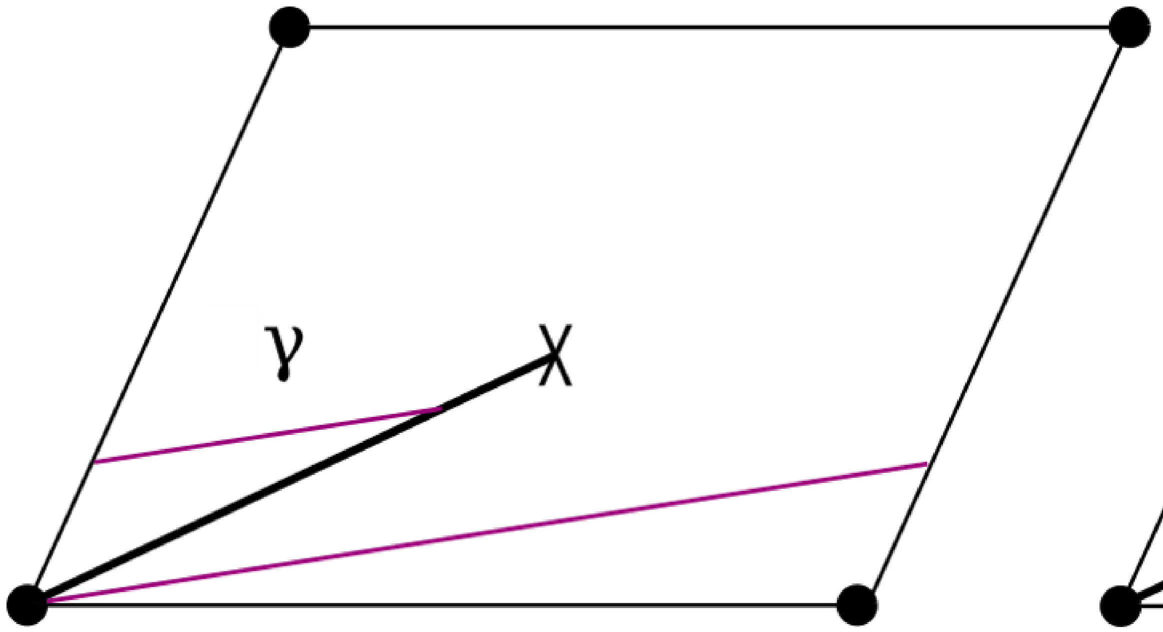
Of special interest is the 1-parameter family

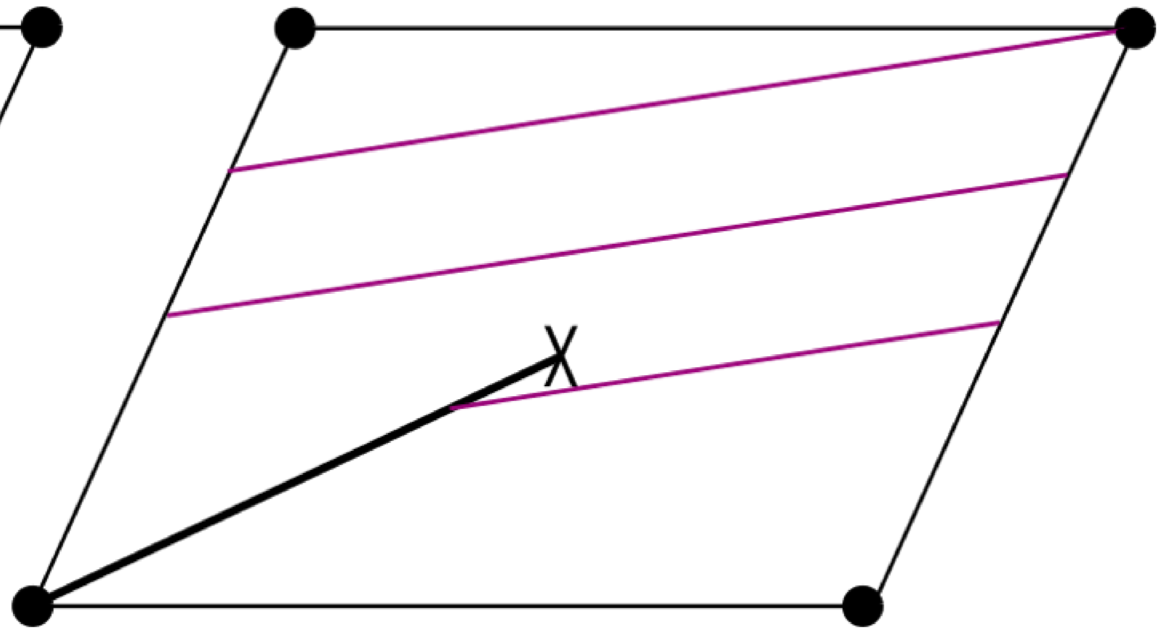
$$\left\{ h_u = \begin{pmatrix} 1 & 0 \\ -u & 1 \end{pmatrix} : u \in \mathbb{R} \right\}$$

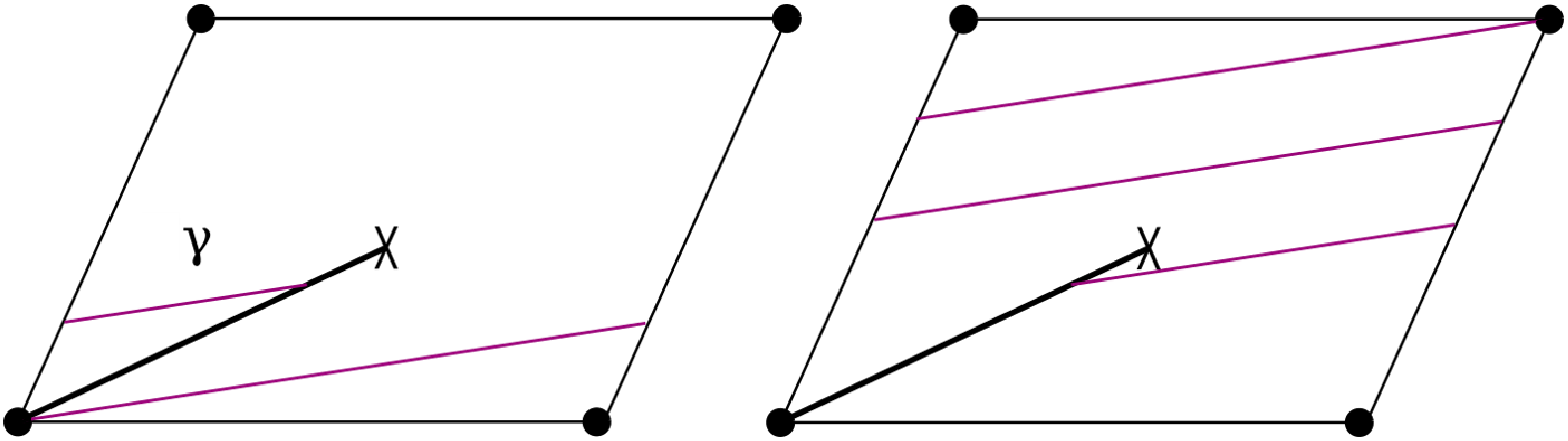
To define the transversal on $\mathcal{E}$ under horocycle flow
we need to introduce certain paths on doubled slit tori



A ***saddle connection*** is a straight-line trajectory starting and ending at a cone type singularity.

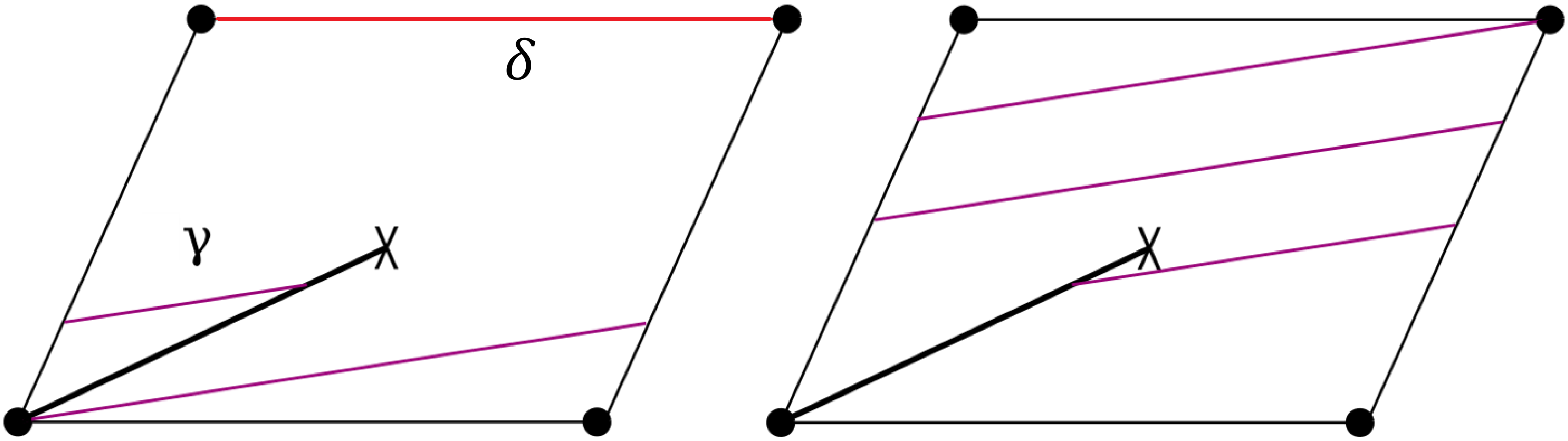




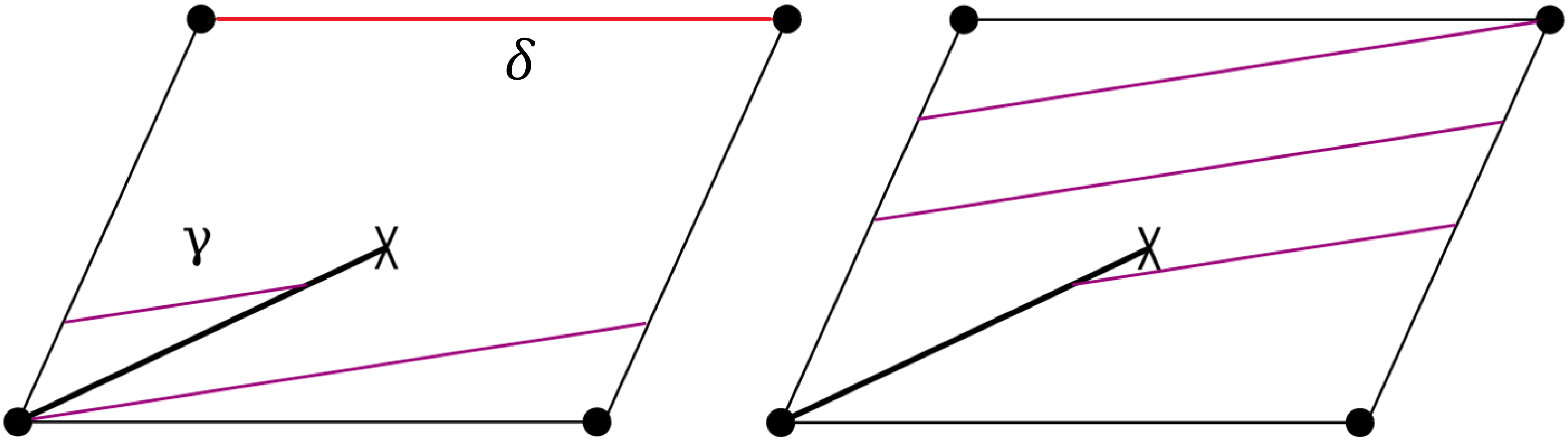


Associated to each saddle connection is the ***holonomy vector***.

$$\textbf{hol}(\gamma) = \binom{4}{1}$$



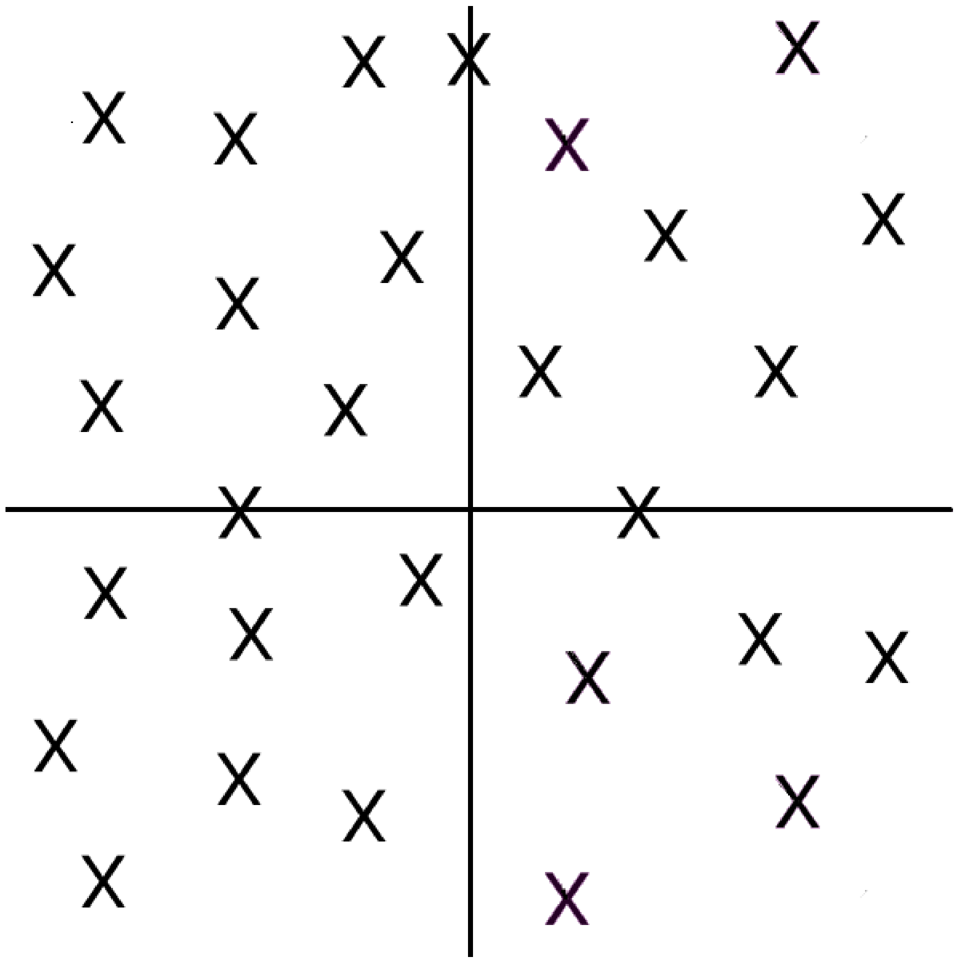
hol(δ) = $\begin{pmatrix}1\\0\end{pmatrix}$



hol(γ) = $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and **hol**(δ) = $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Discreteness

Λ_ω



Let Λ_ω denote the set of all holonomy vectors of a doubled slit torus ω

Veech: Λ_ω is a discrete subset!

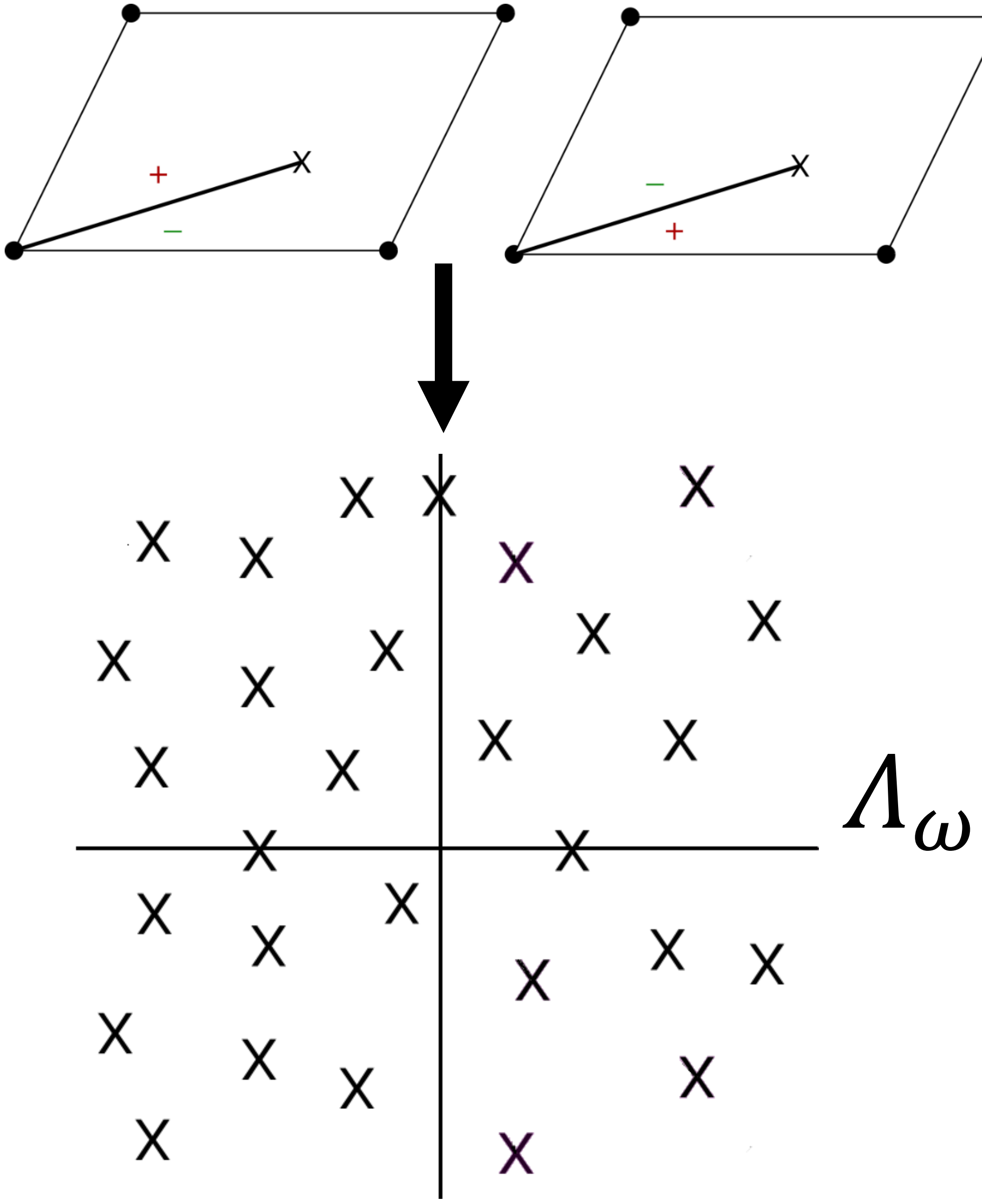
Transversal to horocycle flow on the moduli space of doubled slit tori

Transversal

Consider the ***transversal*** for doubled slit tori

$$\mathcal{W} = \{\omega \epsilon \mathcal{E} | \Lambda_\omega \cap (0,1] \neq \emptyset\}$$

That is, the doubled slit tori that have a *short* horizontal saddle connection.



Theorem (S. 2020)

There is a four-dimensional parameterization of $\mathcal{W}$ and the return time map in these coordinates is given explicitly.

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$$\mathcal{R}(\omega) = \begin{cases} v_2/v_1, \\ (ab)^{-1}, \\ (ab)^{-1} - s, \\ \frac{sa}{\alpha - a}, \\ \frac{a^{-1} - sb}{b + \alpha}. \end{cases}$$

So what?

Guiding philosophy

Questions about a *fixed* translation surface can be understood by considering the dynamics on the space of *all* translation surfaces.

Transversal $\mathcal{W}$ to horocycle flow → Holonomy vectors Λ_ω of a doubled slit torus ω

Guiding philosophy

Transversal $\mathcal{W}$ to horocycle flow

→

(Athreya, Chaika, Cheung, Lelièvre, Work, Uyanik)

Gap distribution of slopes of holonomy vectors Λ_ω of a doubled slit torus ω

(Gap distributions are a measure of randomness of a deterministic set)

Thank you!

Special thanks to:

- Dr. Jayadev Athreya (My advisor)
- Expanding Dynamics: Creative Online Ventures in Dynamics