Title: Metastability for small-random perturbations of a PDE with blow-up

Abstract:

We consider the stochastic PDE $u_t = u_{xx} + u^p + \varepsilon \dot{W}$ with homogenous Dirichlet boundary conditions, where $p > 1$, $\varepsilon > 0$ is a small fixed parameter and $\dot{W}$ stands for space-time white noise. It is well known that the associated deterministic PDE (i.e. $\varepsilon = 0$ in the equation above) admits exactly one asymptotically stable equilibrium and a countable family of unstable equilibria with increasing energy. Furthermore, for certain initial conditions it can be shown that the solution of the deterministic PDE explodes in finite time. We show that, for $p < 5$ and initial conditions in the domain of attraction of the asymptotically stable equilibrium, the solution $X_\varepsilon$ of the SPDE satisfies in the limit as $\varepsilon$ tends to zero the classical description of metastability featured in [1]: the averages of $X_\varepsilon$ remain stable and close to the equilibrium up until the explosion time which, when suitably rescaled, converges in distribution to an exponential random variable. Furthermore, for certain initial conditions in the domain of explosion (and any value of $p > 1$) we show the continuity of the explosion time as $\varepsilon$ tends to zero.

[1] Galves, Antonio; Olivieri, Enzo; Vares, Maria Eulália. Metastability for a class of dynamical systems subject to small random perturbations. Ann. Probab. 15 (1987), no. 4, 1288-1305.