FLACAM 2019

French Latin - American Conference on New Trends in Applied Mathematics
5 - 8 November, Santiago – Chile

http://eventos.cmm.uchile.cl/flacam2019/

Homogenization and the Neumann-Poincaré operator

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Abstract: Nanometric metallic particles may resonate when excited by visible light at particular frequencies. The resulting concentration of electromagnetic energy around the particles proves quite appealing for many applications in opto-electronics. In the small frequency-electrostatic regime, this phenomenon is modeled by a diffusion equation with a piecewise constant conductivity, which is positive in the background dielectric material, but negative inside the particles. The Neumann Poincaré operator is an integral operator that stems from the representation of the solutions to such PDE's using layer potentials : its eigenfunctions are closely related to plasmonic resonances.

We study this integral operator when the homogeneous background medium contains a collection of periodically distributed inclusions (particles) with negative conductivities. We show that, as the period tends to~0, its spectrum converges to a limiting set that consists of two parts, a Bloch spectrum and a boundary-layer spectrum. The former is the union of the spectra of integral operators associated with quasi-periodic resonances, defined on the periodicity cell. The latter corresponds to eigenfunctions localized near the boundary of the macroscopic domain.

If the conductivity inside the inhomogeneities lies outside the spectrum of the periodic Neumann Poincaré operator, we show that bounded sequences of solutions of the corresponding PDE weakly converge to a homogenized limit. The associated effective matrix is defined by a cell-problem of the usual form, as in the case of an elliptic operator. Conversely, if the homogenized source problem is not well-posed, the conductivity inside the inclusions must lie in $\lim_{\varepsilon \to 0} \sigma_\varepsilon$. This cannot happen when the inclusions are strictly contained in the periodicity cells and if the absolute value of their conductivity is sufficiently large.

This is joint work with Charles Dapogny and Faouzi Triki.