FLACAM 2019

French Latin - American Conference on New Trends in Applied Mathematics
5 - 8 November, Santiago – Chile

http://eventos.cmm.uchile.cl/flacam2019/

On conservation laws and multilayer shallow water systems modeling flotation and sedimentation

Raimund Bürger (Universidad de Concepción, Chile)

Abstract: This presentation deals with the theory, numerical methods, and applications of two different first-order quasilinear systems of time-dependent PDEs that describe gravity-driven solid-liquid separation processes. Applications include mineral processing, wastewater treatment, and geophysics, and are related to the Chilean economic and geographic reality.

The first part of the talk deals with the continuous unit operation of flotation that is extensively used in mineral processing, wastewater treatment, and other applications for selectively separating hydrophobic particles (or droplets) from hydrophilic ones, where both are suspended in a viscous fluid. The hydrophobic particles are attached to gas bubbles that are injected and float as aggregates forming a foam or froth at the top that is skimmed. The hydrophilic particles sediment and are discharged at the bottom. The flotation column is described by studying three phases, namely the fluid, the aggregates, and solid particles, in one space dimension. The resulting model is a triangular system of two scalar conservation laws for the aggregates and solids volume fractions with a multiply discontinuous flux. Steady-state solutions that satisfy all jump and entropy conditions are constructed. For industrially relevant steady states, conditions on feed flows and concentrations are established and mapped as "operating charts". A numerical method is formulated on a pair of staggered grids, and is employed for the simulation of the flotation column.

In the second part, a multilayer shallow water approach for the approximate description of polydisperse sedimentation in a viscous fluid is presented. The solid species differ in density and size, segregate, and form areas of different composition. In addition, the settling of particles influences the motion of the ambient fluid. The multilayer shallow water model allows one to determine the spatial distribution of the solid particles, the velocity field, and the evolution of the free surface of the mixture. The final model can be written as a particular multilayer model with variable density where the unknowns are the average velocities and concentrations in each layer, the transfer terms across each interface, and the total mass. An explicit formula of the transfer terms leads to a reduced form of the system. A Harten–Lax–van Leer (HLL)-type path-conservative numerical method is employed to illustrate the coupled polydisperse

sedimentation and flow fields in various scenarios, including sedimentation in a type of basin that is used in the mining industry and in a basin that gives rise to recirculations.

This presentation is based on joint work with Stefan Diehl (Lund University, Sweden), María Carmen Martí (Universitat de València, Spain), Enrique D. Fernández-Nieto (Universidad de Sevilla, Spain), Yolanda Vásquez, and Víctor Osores (both Universidad de Concepción).