FLACAM 2019

French Latin - American Conference on New Trends in Applied Mathematics
5 - 8 November, Santiago – Chile

http://eventos.cmm.uchile.cl/flacam2019/

Plenary talks abstracts

Homogenization and the Neumann-Poincaré operator

Eric Bonnetier (Université Grenoble-Alpes, France)

Nanometric metallic particles may resonate when excited by visible light at particular frequencies. The resulting concentration of electromagnetic energy around the particles proves quite appealing for many applications in opto-electronics. In the small frequency-electrostatic regime, this phenomenon is modeled by a diffusion equation with a piecewise constant conductivity, which is positive in the background dielectric material, but negative inside the particles. The Neumann Poincaré operator is an integral operator that stems from the representation of the solutions to such PDE's using layer potentials : its eigenfunctions are closely related to plasmonic resonances.

We study this integral operator when the homogeneous background medium contains a collection of periodically distributed inclusions (particles) with negative conductivities. We show that, as the period tends to~0, its spectrum converges to a limiting set that consists of two parts, a Bloch spectrum and a boundary-layer spectrum. The former is the union of the spectra of integral operators associated with quasi-periodic resonances, defined on the periodicity cell. The latter corresponds to eigenfunctions localized near the boundary of the macroscopic domain.

If the conductivity inside the inhomogeneities lies outside the spectrum of the periodic Neumann Poincaré operator, we show that bounded sequences of solutions of the corresponding PDE weakly converge to a homogenized limit. The associated effective matrix is defined by a cell-problem of the usual form, as in the case of an elliptic operator. Conversely, if the homogenized source problem is not well-posed, the conductivity inside the inclusions must lie in $\lim_{\varepsilon \to 0} \sigma_\varepsilon$. This cannot happen when the inclusions are strictly contained in the periodicity cells and if the absolute value of their conductivity is sufficiently large.

This is joint work with Charles Dapogny and Faouzi Triki.

On conservation laws and multilayer shallow water systems modeling flotation and sedimentation

Raimund Bürger (Universidad de Concepción-CMM, Chile)

This presentation deals with the theory, numerical methods, and applications of two different first-order quasilinear systems of time-dependent PDEs that describe gravity-driven solid-liquid separation processes. Applications include mineral processing, wastewater treatment, and geophysics, and are related to the Chilean economic and geographic reality.

The first part of the talk deals with the continuous unit operation of flotation that is extensively used in mineral processing, wastewater treatment, and other applications for selectively separating hydrophobic particles (or droplets) from hydrophilic ones, where both are suspended in a viscous fluid. The hydrophobic particles are attached to gas bubbles that are injected and float as aggregates forming a foam or froth at the top that is skimmed. The hydrophilic particles sediment and are discharged at the bottom. The flotation column is described by studying three phases, namely the fluid, the aggregates, and solid particles, in one space dimension. The resulting model is a triangular system of two scalar conservation laws for the aggregates and solids volume fractions with a multiply discontinuous flux. Steady-state solutions that satisfy all jump and entropy conditions are constructed. For industrially relevant steady states, conditions on feed flows and concentrations are established and mapped as "operating charts". A numerical method is formulated on a pair of staggered grids, and is employed for the simulation of the flotation column.

In the second part, a multilayer shallow water approach for the approximate description of polydisperse sedimentation in a viscous fluid is presented. The solid species differ in density and size, segregate, and form areas of different composition. In addition, the settling of particles influences the motion of the ambient fluid. The multilayer shallow water model allows one to determine the spatial distribution of the solid particles, the velocity field, and the evolution of the free surface of the mixture. The final model can be written as a particular multilayer model with variable density where the unknowns are the average velocities and concentrations in each layer, the transfer terms across each interface, and the total mass. An explicit formula of the transfer terms leads to a reduced form of the system. A Harten–Lax–van Leer (HLL)-type path-conservative numerical method is employed to illustrate the coupled polydisperse sedimentation and flow fields in various scenarios, including sedimentation in a type of basin that is used in the mining industry and in a basin that gives rise to recirculations.

This presentation is based on joint work with Stefan Diehl (Lund University, Sweden), María Carmen Martí (Universitat de València, Spain), Enrique D. Fernández-Nieto (Universidad de Sevilla, Spain), Yolanda Vásquez, and Víctor Osores (both Universidad de Concepción).

Phase transitions and symmetry in PDEs

Jean Dolbeault (Université Paris-Dauphine, France)

The question of the symmetry of the solutions of a PDE which is invariant under a transformation is a fascinating question. If a PDE is invariant under rotations, when do we know that solutions, for instance the solutions which minimize the energy, inherit of radial symmetry? This lecture is intended to review some problems and partial answers to these questions, with a special emphasis on problems at the interface of an elliptic point of view, interpreted as the description of stationary solutions, and a dynamical approach based on related equations of evolution.

Eigenvalues and approximation of (Cantor) dynamical systems: 20 years of mathematics with the CMM and Chile

Fabien Durand (Université Picardie Jules Verne, France)

In this talk we will present a well-known and nice way to approximate dynamical systems from an ergodic and topological point of view.

We will then use these approximations to understand the existence of continuous eigenfunctions for the Koopman operator. This will be illustrated through many examples.

This will give me the opportunity to show a small part of the strong mathematical cooperation between France and Chile.

This much older cooperation gave rise to the association of the CNRS with the CMM in 2000, almost 20 years ago.

Large graphs and other combinatorial structures

Yoshiharu Kohayakawa (U. de São Paulo, Brazil)

A fundamental result in graph theory, Szemerédi's regularity lemma, tells us that large graphs can be described with a small amount of data when we are interested in certain types of problems. When we have a sequence of graphs whose orders tend to infinity, if their subgraph statistics are consistent, one can define a continuous object that is the limit of the sequence. These are the celebrated graph limits of Lovász and Szegedy. In this talk, we shall introduce and discuss some basic facts in these two approaches to the study of large graphs. Some other large combinatorial structures will also be considered.

Diffusion and competition in population and gender dynamics

Salomé Martínez (Universidad de Chile-CMM, Chile)

Reaction-diffusion models have been widely used to study fundamental questions in population dynamics. This type of partial differential equations provides a way to translate local assumptions regarding the movement, growth and interactions for the individuals of a species, into global features of the population. Thus, reaction-diffusion models provide a theoretical framework for questions such as the persistence of a species, invasions, and coexistence of populations. Mathematical tools from non-linear analysis and dynamical systems can be used to study the consequences of population characteristics have in the long-term dynamics. We will discuss how the relationship between population dispersal strategies, environmental factors and competition affects the persistence and coexistence of two species.

In this talk we will also explore issues related to the persistence and dispersal of women in a STEM environment, in which they account for less than 30% of the population. We will discuss the strategies which have been key for persistence, allowing women in STEM to grow and thrive through the formation and strengthening of networks and alliances. In particular, we will discuss the process that led to the creation of the Direction for Diversity and Gender, the first of this kind in a Faculty of Sciences, Math, and Engineering in Chile, and some research projects that the direction in pursuing.

Adaptation to a gradual environment - Research of lineages

Sylvie Méléard (École Polytechnique, France)

Directional environmental changes, such as caused by climate warming, imposes strong selection on many living organisms, which need to evolve fast enough to keep track of their changing environment. We introduce a quantitative genetics model exploring this question and consider some phenotypic trait subject to stabilizing selection around some optimal phenotype, which value is shifted continuously through time. We construct the stochastic individual based model and its deterministic PDE approximation for which we exhibit a stationary distribution. Assuming now that the density profile stays at this equilibrium, we are interested in the lineage of an individual uniformly sampled at a fixed time. Our aim is to capture the distribution of the initial value of this trajectory and to exhibit a biais in the distribution. We use a spinal approach classical for branching processes. We will also give the equation of such lineage. It's a work in progress with V. Calvez, B. Henry, F. Patout and C.V. Tran.

Defining shadow prices in industrial mathematics

Claudia Sagastizábal (IMECC Unicamp, Brazil)

In Linear Programming, the meaning of dual variables as shadow prices is well-known: Lagrange multipliers signal the marginal effect of perturbing the constraint set of an optimization problem. When the constraint expresses satisfaction of consumer demand for goods and services, the multiplier measures the willingness to pay for one more unit of the item. For an environmental constraint limiting the emission of greenhouse gases in some industrial process, the multiplier can be seen as revealing the price of decarbonizing the process under consideration.

The concept is straightforwardly applicable to Nonlinear Programming but not to optimization problems with mixed-integer variables. Industrial processes often involve binary decisions, such as turning on or off some production unit, which amounts to having some variable components taking the values 0 or 1. In this setting, the optimization problem does not have Lagrange multipliers and the useful notion of shadow price is not available.

We discuss how to circumvent this drawback by means of Variational Analysis. All along the presentation, simple examples are used to illustrate the interest of the approach. Credit to various co-authors will be given during the talk.