GRAPH THEORY IN THE ANDES

ABSTRACTS

MONDAY

Chair: Marcos Kiwi

Paul Seymour 9:05-9:55

Excluding holes

A "hole" in a graph is an induced subgraph which is a cycle of length 3. The perfect graph theorem says that if a graph has no odd holes and no odd antiholes (the complement of a hole), then its chromatic number equals its clique number; but unrestricted graphs can have clique number two and chromatic number arbitrarily large. There is a nice question half-way between them – for which classes of graphs is it true that a bound on clique number implies some (larger) bound on chromatic number? Call this being "chi-bounded".

Gyarfas proposed several conjectures of this form in 1985, and recently there has been significant progress on them. For instance, he conjectured:
– graphs with no odd hole are chi-bounded (this is true);
– graphs with no hole of length > 100 are chi-bounded (this is true);
– graphs with no odd hole of length > 100 are chi-bounded; this is still open but true for triangle-free graphs.

We survey this and several related results. This is joint with Alex Scott and partly with Maria Chudnovsky.

Matt DeVos 10:30-10:55

TBA

Sang-il Oum 11:00-11:25

Improper coloring and the odd Hadwiger's conjecture

Gerards and Seymour conjectured a strengthening of Hadwiger's conjecture, stating that every graph with no K_t odd minor admits a partition of its vertex set into $t-1$ independent sets. We prove that there is a function f such that for $t \geq 2$, every graph with no K_t odd minor admits a partition of its vertex set into $7t-10$ sets, each inducing a subgraph of maximum degree at most $f(t)$. As a corollary, we deduce that every graph with no K_t odd minor admits a partition of its vertex set into $28t-40$ sets, each inducing a subgraph having no large components. This corollary improves the bound by Kawarabashi (2008) who showed that every such graph admits a partition of its vertex set into $496t$ sets., each inducing a subgraph having no large components.

This is a joint work with Dong Yeap Kang.

Jayme Szwarcfiter 17:00-17:25

On the computation of graph parameters related to graph convexities

A *graph convexity* is a pair $(G, \mathcal{C})$, where G is a finite graph with vertex $V(G)$ and $\mathcal{C}$ a family of subsets of $V(G)$ satisfying $\emptyset, V(G) \in \mathcal{C}$ and being closed under intersections. The sets $C \in \mathcal{C}$ are called *convex sets*. The most common graph convexities are those whose convex sets are defined through special paths of the graph. Among them the most prominent are the *geodesic convexity*, where $\mathcal{C}$ is closed under taking shortest paths, the *monophonic convexity*, where $\mathcal{C}$ is closed under induced subgraphs and the P_3 convexity, whose convex sets are closed under pairs of common neighbors. In special, the latter is closely related to some well studied graph processes, as percolation and conversion processes. Among the parameters related to graph convexities, we can mention the *interval number, convexity number, hull number, Helly number, Carathéodory number, Radon number and rank.* In this talk, we describe complexity results related to the computation of the rank of a graph. These include characterizations, hardness results, polynomial-time algorithms and bounds.

Luciano Grippo 17:30-17:55

Given a graph $G = (V, E)$, a set X of vertices is called convex if the graph induced by X, contains all shortest paths between any two of its vertices. In this talk we will focus on the notion of a convex p-partition of a graph, that is, a partition of the vertex set into p convex sets. For instance, any graph on n vertices containing a matching of size m has a convex $(n - m)$-partition, and trivially, any graph has a convex 1-partition. Deciding whether a graph has a convex p-partition, for fixed p greater or equal than 2, is NP-complete for arbitrary graphs, and linear time solvable for cographs. Also, chordal graphs have convex p-partitions for all p greater or equal than 1.

It was conjectured by Pelayo that also for bipartite graphs, it should be NP-complete to decide whether they have a convex p-partition. We show that, for any fixed p, this is not the case (unless $P = NP$). More precisely, we prove that for every p, all convex p-partitions of a bipartite graph can be enumerated in polynomial time. This extends a recent result of Glantz and Meyerhenke, who prove the same for the case $p = 2$. They also showed that all convex 2-partitions of a planar graph can be found in polynomial time

TUESDAY

CHAIR: JOSÉ SOTO

Alex Scott 9:00-9:50

Holes in graphs of large chromatic number

In 1985, Gyarfas made three well-known conjectures on holes in graphs of large chromatic number. Two of these conjectures have now been proved. Paul Seymour's talk will survey these and related results. In this talk, we will describe some of the techniques used in the proofs.

Joint work with Maria Chudnovsky and Paul Seymour.

Jozef Skokan 15:00-15:25

The Multicolour Ramsey Number of a Long Odd Cycle

For a graph G, the k-colour Ramsey number $R_k(G)$ is the least integer N such that every k-colouring of the edges of the complete graph K_N contains a monochromatic copy of G. Let C_n denote the cycle on n vertices. We show that for fixed $k > 3$ and n odd and sufficiently large, $R_k(C_n) = 2^{k-1}(n-1)+1$. This generalises a result of Kohayakawa, Simonovits and Skokan and resolves a conjecture of Bondy and Erdős for large n. We also establish a surprising correspondence between extremal k-colourings for this problem and perfect matchings in the hypercube Q_k. This allows us to in fact prove a stability-type generalisation of the above. The proof is analytic in nature, the first step of which is to use the Regularity Lemma to relate this problem in Ramsey theory to one in convex optimisation.

This is joint work with Matthew Jenssen.

Louis DeBiasio 15:30-15:55

Monochromatic partitioning of non-complete graphs

Haxell and Kohayakawa proved that in every r-coloring of the edges of a complete graph on sufficiently large vertex set V, there exists at most r vertex disjoint monochromatic components which together contain all of the vertices of V. This gave a partial result towards a conjecture of Erdos, Gyarfas, and Pyber, which itself was a strengthening of the famous Ryser's conjecture (where a cover with r-1 – not necessarily vertex disjoint – monochromatic components is desired).

We will consider variations on this theme where the underlying graph is not complete, but has sufficiently large minimum degree, is pseudorandom in some sense, or is the random graph G(n,p); and variations where the monochromatic components have a specific structure such as paths or cycles. The main objective will be to survey recent results in the area and present some open problems.

Based on joint work with Deepak Bal and Luke Nelsen.

Richard Lang 16:00-16:25

Monochromatic Cycle Partitioning

Given an arbitrary edge-colouring of the complete graph K_n with r colours, how many monochromatic cycles are needed to partition its vertices? (Here single edges and vertices count as cycles as well.)

It is not trivial that this number is independent of the size of K_n. This, however, was shown in a landmark paper by Erdős, Gyárfás and Pyber in 1991. They proved that K_n can be partitioned into $25r^2 \log r$ monochromatic cycles and conjectured that this can be brought down to (optimal) r cycles. The conjecture has been recently disproved, but it is believed that it was not too far off the mark.

In this talk I will give a short introduction to the topic and present some open problems.

WEDNESDAY

Chair: Cristina Fernandes

Mathias Schacht 9:00-9:50

Turán-type problems for quasirandom hypergraphs

Extremal problems for hypergraphs concern the maximum density of large hypergraphs H that do not contain a copy of a given hypergraph F. Estimating the so-called Turán-densities is a central problem in combinatorics. However, despite a lot of effort precise estimates are only known for very few hypergraphs F.

We consider a variation of the problem, where the large hypergraphs H satisfy additional quasirandom conditions on its edge distribution. We present recent progress based on joint work with Reiher and Rödl for 3-uniform hypergraphs. In particular, we established a computer-free proof of a recent result of Glebov, Král' and Volec on the Turán-density of so-called weakly quasirandom hypergraphs not containing the 3-uniform hypergraph with three edges on four vertices.

Carlos Hoppen 12:00-12:25

Graphs with a large number of edge-colorings avoiding a given pattern

We consider a problem that was motivated by a question raised by Erdős and Rothschild in the 1970s. They asked for n-vertex graphs that admit the largest number of edge-colorings avoiding a monochromatic copy of some fixed graph F, and conjectured that the n-vertex k-partite Turán graph admits the largest number of 2-colorings of K_{k+1}. This was confirmed by Yuster in the case of triangles and by Alon, Balogh, Keevash and Sudakov in the remaining cases, and led to the study of other forbidden graphs and coloring patterns. In this talk, we shall present some recent results in this direction.

Hiep Hàn 12:30-12:55

Maximum number of colourings without monochromatic Schur triples

Abstract: We study subsets of finite abelian groups that maximize the number of r-colourings without monochromatic Schur triples, i.e. triples of the form (a, b, c) such that $a + b = c$. For $r = 2, 3$ and a large class of abelian groups, we show that the maximum is achieved only by largest sum-free sets. For $r > 3$ this phenomenon does not persist and the problem becomes significantly harder. We resolve the problem for abelian groups of even order and $r = 4, 5$.

Joint work with Andrea Jimenez.

<u>THURSDAY</u>

Chair: José Zamora

Janos Pach 15:30-16:20

The Converse of Tarski's Plank Problem

A plank is the part of space between two parallel hyperplanes. The width of a convex body is the smallest width of a plank containing it. According to Tarski's Plank Problem (Bang's theorem), the total width of any system of planks that cover a d-dimensional convex body C is at least the width of C. We discuss the following conjecture of Makai and myself. Let $P_1, P_2, \ldots$ be a sequence of planks in the d-dimensional space with width $w_1, w_2, \ldots$, respectively. Then each P_i has translate such that the union of these translates covers the whole space if and only if the series $w_1 + w_2 + \ldots$ is divergent. This is known to be true in the plane, but open for every $d > 2$.

Joint work with A. Kupavskii.

Flavia Bonomo 17:00-17:25

Three-Coloring and List Three-Coloring Graphs without Induced Paths on Seven Vertices

In this talk, we will present an algorithm to 3-color an n-vertex graph without induced paths on seven vertices in polynomial time. This problem was posed by Schiermeyer, Randerath and Tewes in 2002. We solve indeed a more general list-coloring problem, where every vertex is assigned a list of colours which is a subset of $\{1, 2, 3\}$. The complexity of the algorithm is is $O(n^{23})$. We further present a simpler and faster algorithm for the list three-coloring problem when the graph is also triangle-free. The complexity of the second algorithm (for $\{P7, triangle\}$-free graphs) is $O(n^7)$, and if G is bipartite, it improves to $O(n^4)$.

Joint work with Maria Chudnovsky, Peter Maceli, Oliver Schaudt, Maya Stein and Mingxian Zhong

Florencia Fernández 17:30-17:55

On unit interval graphs with integer endpoints

We study those unit interval graphs having a model with intervals of prescribed integer length. We present a structural result for this subclass which leads to a quadratic–time recognition algorithm of it, giving as positive certificate a model of minimum total length and as negative certificate a forbidden induced subgraph. We also present a quadratic–time algorithm to determine, given a unit interval graph, the minimum length of its intervals in order to have an integer interval model. Joint work with G. Durán, L. Grippo, F. Oliveira, J. Szwarcfiter.

FRIDAY

CHAIR: PIA MAZZOLENI

Dieter Mitsche 9:00-9:25

On-line list coloring of random graphs

In this talk, we consider the on-line list colouring of binomial random graphs $G(n,p)$. We show that the on-line choice number of $G(n,p)$ is asymptotically almost surely asymptotic to the chromatic number of $G(n,p)$, provided that the average degree $d = p(n-1)$ tends to infinity faster than $(\log\log n)^{1/3}(\log n)^2 n^{2/3}$. For sparser graphs, we are slightly less successful; we show that if $d > (\log n)^{2+\epsilon}$ for some $\epsilon > 0$, then the on-line choice number is larger than the chromatic number by at most a multiplicative factor of C, where $C \in [2,4]$, depending on the range of d. Also, for $d = O(1)$, the on-line choice number is by at most a multiplicative constant factor larger than the chromatic number.

Joint work with Alan Frieze, Xavier Perez-Gimenez and Pawel Pralat.

Henning Bruhn 9:30-9:55

Erdős-Pósa properties with roots

Any graph contains k disjoint cycles, or a small hitting set, ie, a vertex set of size about $4k\log k$ whose removal results in an acyclic graph—this is the contents of the classic Erdős-Pósa theorem. The theorem has been extended in various ways: to long cycles, to graphs having a fixed planar minor etc.

In the talk I will discuss two recent generalisations. The Erdős-Pósa theorem remains true for:

1) long rooted cycles
2) certain rooted minors

The talk is based on joint work with Felix Joos and Oliver Schaudt.

Martín Safe 12:00-12:25

On the hereditary (p,q)-Helly property of hypergraphs, cliques, and bicliques

We prove several characterizations of hereditary (p,q)-Helly hypergraphs, including one by minimal forbidden partial subhypergraphs, and show that the recognition of hereditary (p,q)-Helly hypergraphs can be solved in polynomial time for fixed p but is co-NP-complete if p is part of the input. We also give several characterizations of hereditary (p, q)-clique-Helly graphs, including one by forbidden induced subgraphs, and prove that the recognition of hereditary (p, q)-clique-Helly graphs can be solved in polynomial time for fixed p and q but is NP-hard if p or q is part of the input. We prove similar results for hereditary (p, q)-biclique-Helly graphs.

Ivan Rapaport 12:30-12:55

Solving the Induced Subgraph problem in the randomized multiparty simultaneous messages model

We study the message size complexity of recognizing, under the broadcast congested clique model, whether a fixed graph H appears in a given graph G as a minor, as a subgraph or as an induced subgraph. The n nodes of the input graph G are the players, and each player only knows the identities of its immediate neighbors. We are mostly interested in the one-round, simultaneous setup where each player sends a message of size $O(logn)$ to a referee that should be able then to determine whether H appears in G. We consider randomized protocols where the players have access to a common random sequence. We completely characterize which graphs H admit such a protocol.

This is joint work with Jarkko Kari, Martin Matamala and Ville Salo.