Inverse Problems for the Physical Sciences 2024

Program, titles and abstracts

Puerto Varas, Chile, 15-19 January 2024

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- Gunther Uhlmann (University of Washington, US)
- Matti Lassas (University of Helsinki, Finland)
- Eric Bonnetier (Université Grenoble-Alpes, France)

Local Organizing Committee

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Program

TIME	MONDAY 15TH	TUESDAY 16TH	WEDNESDAY 17TH	THURSDAY 18TH
9:00-9:30	Registration and opening words			
9:30-10:30	Andras Vasy	Matti Lassas	Plamen Stefanov	Eric Bonnetier
10:30-11:00	coffee break	coffee break	coffee break	coffee break
11:00-12:00	Sean Holman	Giovanni Alberti	Lauri Oksanen	Sebastian Acosta
12:00-13:00	Axel Osses	John Schotland	Laurent Seppecher	Spyridon Filippas
13:00-15:00	lunch	lunch	lunch	lunch
15:00-16:00	Ravi Prakash	Mircea Petrache	Esteban Paduro	Gunther Uhlmann
16:00-16:30	Medet Nursultanov	Sebastian Muñoz	coffee break	coffee break
16:30-17:00	coffee break	coffee break		
17:00-17:30	Hugo Carrillo	Alex Imba		
19:00-22:00			Banquet	

Invited Speakers

PSEUDODIFFERENTIAL APPROXIMATIONS OF ULTRASOUND WAVES FOR BIOMEDICAL APPLICATIONS

Sebastian Acosta

Pediatrics-Cardiology Predictive Analytics Laboratory, Baylor College of Medicine

Computational simulations are playing an increasingly important role to improve therapeutics and diagnostics using ultrasound waves. To unleash the full potential of computer simulations, the computational method must strike the right balance between accuracy and speed. We develop a fast pseudodifferential method that takes advantage of the geometric flow of acoustic energy to efficiently handle the highly oscillatory nature of ultrasound waves. Simultaneously, the method accurately incorporates refraction, reflection and attenuation imposed by realistic models of biological media. As a result, simulations of ultrasound wave propagation can be executed orders of magnitude faster than using conventional approaches. Some numerical results will be presented, and limitations discussed. This is joint work with Benjamin Palacios (Pontificia Universidad Católica de Chile) and Jesse Chan (Rice University).

ON THE SAMPLE COMPLEXITY OF INVERSE PROBLEMS

Giovanni Alberti

Department of Mathematics, University of Genoa

Many inverse problems are modelled by integral or partial differential equations, including, for instance, the inversion of the Radon transform in computed tomography and the Calder ´on problem in electrical impedance tomography. As such, these inverse problems are intrinsically in- finite dimensional and, in theory, require infinitely many measurements for the reconstruction. In this talk, I will discuss recovery guarantees with finite measurements, and with explicitly estimates on the sample complexity, namely, on the number of measurements. These results use methods of sampling theory and compressed sensing, and work under the assumption that the unknown either belongs to a finite-dimensional subspace/submanifold or enjoys sparsity properties. I will consider both linear problems, such as the sparse Radon transform, and nonlinear problems, such as the Calder´on problem and inverse scattering.

A similar issue arises when applying machine learning methods for solving inverse problems, as for instance to learn the regularizer, which may depend on infinitely many parameters. I will present sample complexity results on the size of the training set, both in the case of generalized Tychonov regularization, and with ℓ^1-type penalties.

This talk is based on a series of joint works with Á. Arroyo, E. De Vito, A. Felisi, M. Lassas, L. Ratti, M. Santacesaria, S. Sciutto and S. I. Trapasso.

PERTURBATION OF AN ELLIPTIC EQUATION BY CHANGING THE TYPE OF BOUNDARY CONDITION ON A SMAL SET

Eric Bonnetier

Institut Fourier, Université Grenoble Alpes

Asymptotic expansions of the solution to an elliptic PDE in the presence of inclusions of small size have found successful applications in inverse problems, as a means to detect inhomogeneities from boundary measurements in a robust way.

In this talk, instead of perturbations in the bulk, we consider perturbations of the boundary conditions. For instance a homogeneous Neumann condition may be replaced by a homogeneous Dirichlet condition on a “small" set ω_ε, or vice-versa. We characterize the first term in the asymptotic expansion of the solution, in terms of the relevant measure of smallness of ω_ε, and give explicit examples when ω_ε is a small surface ball in $\mathbb{R}^d$, $d = 2,3$.

This is joint work with Charles Dapogny and Michael Vogelius.

NEUTRON TRANSMISSION STRAIN IMAGING, THE LONGITUDINAL RAY TRANSFORM AND PDES

Sean Holman

Department of Mathematics, The University of Manchester

I will discuss neutron transmission strain imaging which attempts to measure the residual strain field inside a polycrystalline object, such as a metal, by transmission of an energy resolved beam of neutrons. In this application, it is possible to determine the longitudinal ray transform (LRT) of the strain field inside the object leading to an integral geometry problem for reconstruction. As is well known, from the LRT it is only possible to reconstruct the solenoidal part of a tensor field, but by adding the physical equilibrium condition satisfied by the strain (a PDE) we can determine more. This talk is based on joint work with Chris Wensrich, Matias Courdurier, Bill Lionheart, Anna Polyakova, and Ivan Svetov which was begun during a program at the Isaac Newton Institute.

MAPPING PROPERTIES OF NEURAL NETWORKS, NEURAL OPERATORS, AND INVERSE PROBLEMS

Matti Lassas

Department of Mathematics and Statistics, University of Helsinki

We will consider mapping properties of neural networks and neural operators which are infinite dimensional generalizations of neural networks. In particular, we consider the injectivity of neural networks and universal approximation property of injective neural networks. In addition, we study approximation of probability measures using neural networks that are compositions of invertible flow networks and injective layers and present applications in inverse problems. We also discuss the similarities of neural operators and pseudodifferential operators.
The results have been done in collaboration with M. de Hoop, I. Dokmanic, T. Furuya, P. Pankka and M. Puthawala.

INVERSE PROBLEM FOR YANG-MILLS-HIGGS FIELDS

Lauri Oksanen

Department of Mathematics and Statistics, University of Helsinki

We show that the Yang-Mills potential and Higgs field are uniquely determined (up to the natural gauge) from source-to-solution type data associated with the classical Yang-Mills-Higgs equations in the Minkowski space. We impose natural non-degeneracy conditions on the representation for the Higgs field and on the Lie algebra of the structure group which are satisfied for the case of the Standard Model. Our approach exploits non-linear interaction of waves to recover a broken non-abelian light ray transform of the Yang-Mills field and a weighted integral transform of the Higgs field. The talk is based on joint work with Chen Xi, Matti Lassas, and Gabriel Paternain.

SOME INVERSE PROBLEMS FOR PDE's IN HEART MUSCLE FIBER IDENTIFICATION

Axel Osses

Department of Mathematical Engineering, Universidad de Chile

I present some exact and approximate solutions of weak harmonic maps in two and three dimensions for the nonlinear Frank-Oseen equations from liquid crystal theory and their possible applications in the framework of non-invasive identification of the direction of muscle fibers in the human heart, as well as their striking similarity to certain structures in the field of astrophysics. It is a collaborative work with Nicolás Barnafi (Center for Mathematical Modeling, U. de Chile).

EQUIVARIANT NEURAL NETWORKS AND GEOMETRIC RECONSTRUCTION PROBLEMS

Mircea Petrache

Department of Mathematics, Pontificia Universidad Católica de Chile

Consider a finite metric space, with in mind the important case of a point cloud in $\mathbb{R}^d$. The question of what is the lowest-complexity isometry invariant encoded by functions of distances has produced a series of long-standing open questions. Recently this question has crossed over to the field of Deep Learning due to the relevance to accuracy and stability issues in learning tasks for chemistry with symmetry (isometry) equivariance constraints. After an introduction on general learning theory and of neural networks with symmetries, I will present recent work from NeurIPS2023 on the above problems. The first work, joint with Shubhendu Trivedi, concerns a recent theoretical formulation of the basic approximation-generalization tradeoff for Equivariant Neural Networks. In the second result, obtained in our group at PUC Chile, we formulate a complete isometry invariant for point clouds in Euclidean spaces related to Graph Neural Networks models. For this latter result we show tight bounds for the optimal complexity of complete metric invariants, within the scale given by so-called geometric Weisfeiler-Lehman isomorphism tests.

THE MONOTONICITY PRINCIPLE FOR NONLINEAR ELECTRIC CONDUCTIVITY TOMOGRAPHY

Ravi Prakash

Department of Mathematics, Universidad de Concepción

We treat an inverse electrical conductivity problem which deals with the reconstruction of nonlinear electrical conductivity starting from boundary measurements in steady currents operations. In this framework, a key role is played by the Monotonicity Principle, which establishes a monotonic relation connecting the unknown material property to the (measured) Dirichlet-to-Neumann operator (DtN). Monotonicity Principles are the foundation for a class of non-iterative and real-time imaging methods and algorithms. In this talk, we prove that the Monotonicity Principle for the Dirichlet Energy in nonlinear problems holds under mild assumptions. Then, we show that apart from linear and p- Laplacian cases, it is impossible to transfer this Monotonicity result from the Dirichlet Energy to the DtN operator. To overcome this issue, we introduce a new boundary operator, identified as an Average DtN operator. This talk is based on the joint work with G. Piscitelli, A. Corbo Esposito, L. Faella and A. Tamburrino: The Monotonicity Principle for Nonlinear Electrical Conductivity, Published in Inverse Problems, available online at https: //iopscience.iop.org/ article/10.1088/1361-6420/abb51c.

INVERSE PROBLEMS FOR NONLOCAL PDES WITH APPLICATIONS TO QUANTUM OPTICS

John Schotland

Department of Mathematics, Yale University

Abstract: We propose a method to reconstruct a scattering medium from single photon measurements. The method is based on the solution to an inverse problem for a nonlocal PDE. Such problems arise in the study of cold atom systems in quantum optics.

NON-CONVEX INTERFEROMETRIC INVERSION FOR SPARSE WAVE SOURCES RECOVERY IN COMPLEX MEDIA

Laurent Seppecher

Institut Camille Jordan, École Centrale de Lyon

In non-homogeneous media with unknown wave speed, it could be difficult to recover sources from measurements of the wave field at distant receivers. The Kirchhoff migration may fail, as well as the full source inversion from a least squares approach. This is mainly due to the large dephasing occurred by the wave speed variations that especially affect the high frequency data.

In the regime of smoothly varying media, it is known that the cross correlations between measurements at nearby receivers remain much more stable than linear measurements of the field. The interferometric inversion aims at recovering the source directly from some of the cross-products called the interferometric data. This is a challenging, non-convex, quadratic problem.

In this talk, we will discuss the conditions for the interferometric inversion to be well-posed, up to a constant phase shift, and provide new recovery estimates from interferometric data. We will also see that under the same conditions, and despite the non-convexity of the problem, a Wirtinger flow descend applied on the interferometric misfit functional "locally" converges to the solution. Finally, we will see, in numerical examples, that this approach outperforms by far classical methods to recover sparse sources in slowly varying environment.

THE INVERSE BACK-SCATTERING PROBLEM FOR TIME DEPENDENT POTENTIALS

Plamen Stefanov

Department of Mathematics, Purdue University

We study the inverse problem of determining a time-dependent potential in the wave equation from back-scattering data. We show that if the potential is small enough, compactly supported in the x-variable only, it is stably recoverable. In particular, singularities moving faster than light are visible as well. We reveal an interesting relation between this problem and the Radon transform in timespace of integrating over hyperplanes of codimension two. We are partly inspired by a previous work by Krishnan, Rakesh and Senapati about the recovery of such a potential from near field scattering data where the time is restricted to a finite interval; but the method is different. This is a joint work with Medet Nursultanov and Lauri Oksanen.

SOME OPEN INVERSE PROBLEMS

Gunther Uhlmann

Department of Mathematics, University of Washington

We will discuss several open problems in the theory of inverse problems.

THE FEYNMAN PROPAGATOR AND SELF-ADJOINTNESS

András Vasy

Department of Mathematics, Stanford University

In this talk I will discuss the Feynman and anti-Feynman inverses for wave operators on certain Lorentzian manifolds; these are two inverses which from a microlocal analysis perspective are more natural than the standard causal (advanced/retarded) ones. For instance, for the spectral family of the wave operator, these are the natural inverses when the spectral parameter is non-real. Indeed, I will explain that these connect to the self-adjointness of the wave operator, and the positivity properties that follow.

Young Speakers

TBD

Hugo Carillo

Instituto de Ingeniería Matemática y Computacional, Pontificia Universidad Católica de Chile

ON UNIQUE CONTINUATION FOR WAVES IN SINGULAR MEDIA

Spyridon Filippas

Department of Mathematics and Statistics, University of Helsinki

The problem of unique continuation consists in recovering the whole wave from a partial observation and has applications to control theory and inverse problems. After presenting some fundamental results of the theory we will explain how one can prove a logarithmic stability result for wave operators whose metric exhibits a jump discontinuity across an interface. We make no assumption about the sign of the jump or the geometry of the interface. The key ingredient of our proof is a local Carleman inequality near the interface. Using a propagation argument, we derive then a global stability estimate.

TBD

Alex Imba

Department of Mathematics, Universidad Técnica Federico Santa Maria

BOUNDARY AND SCATTERING RIGIDITY FOR MP-SYSTEMS

Sebastián Muñoz-Thon

Department of Mathematics, Purdue University

An $\mathscr{MP}$-system consist of a compact Riemannian manifold with boundary, endowed with a magnetic field and a potential. In this talk, I will focus on rigidity for simple $\mathscr{MP}$-systems. I will show that the knowledge of the boundary action function or the scattering relation at one energy level determines the $\mathscr{MP}$-system (up to a suitable gauge). By studying the linearized problem, I will show that there exists a generic set of simple $\mathscr{MP}$-systems, such that any two MP- systems close to an element in it and having the same boundary action function, must be gauge equivalent. If time allows, I will discuss some work in progress regarding scattering rigidity in the stationary case.

INVERSE PROBLEM ON MANIFOLDS WITH DISJOINT DATA

Medet Nursultanov

Department of Mathematics and Statistics, University of Helsinki

We consider the inverse problem to determine a smooth compact Riemannian manifold (M, g) from a restriction of the source-to-solution operator, $\Lambda_{\mathcal{S},\mathcal{R}}$, for the wave equation on the manifold. Here, $\mathcal{S}$ and $\mathcal{R}$ are open sets on M, and $\Lambda_{\mathcal{S},\mathcal{R}}$ represents the measurements of waves produced by smooth sources supported on $\mathcal{S}$ and observed on $\mathcal{R}$. We emphasize that $\overline{\mathcal{S}}$ and $\overline{\mathcal{R}}$ could be disjoint. We demonstrate that $\Lambda_{\mathcal{S},\mathcal{R}}$determines the manifold (M, g) uniquely under the following spectral bound condition for the set $\mathcal{S}$: There exists a constant $C > 0$ such that any normalized eigenfunction $\phi$of the Laplace-Beltrami operator on (M, g) satisfies

$$1 \leq C \|\phi|_{\mathcal{S}}\|_{L^2(\mathcal{S})}.$$

We note that, for the Anosov surface, this spectral bound condition is fulfilled for any non-empty open subset S. Moreover, we solve the analogue of this problem for the heat equation, by showing that the source-to-solution maps for the heat and wave equations determine each other. (joint work with Matti Lassas, Lauri Oksanen, Lauri Ylinen)

TBD

Esteban Paduro

Millennium Nucleus For Applied Control and Inverse Problems, Pontificia Universidad Católica de Chile