Summer School 2026

Inverse Problems for the Physical Sciences

Program

Puerto Varas, Chile, 5-8 January 2026

Organizing Committee

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- Universidad de Concepción, Chile
- University of Washington, Seattle, US

Program

Time	Monday 5th	Tuesday 6th	Wednesday 7th	Thursday 8th
9:00 - 10:00	Registration and opening words	Mini course 5	Mini course 5	Mini course 5
10:00 - 10:30	Coffee break	Coffee break	Coffee break	Coffee break
10:30 - 11:30	Mini course 1	Mini course 1	Mini course 1	Talk 5
11:45 - 12:45	Mini course 2	Poster session	Mini course 2	Mini course 2
12:45 - 15:00	Lunch	Lunch	Lunch	Lunch
15:00 - 16:00	Mini course 3	Mini course 3	Talks 3 & 4	Mini course 3
16:15 - 17:15	Mini course 4	Talks 1 & 2	Mini course 4	Mini course 4
				Closing words

Mini Courses

1. **Mathematics of X-ray Computed Tomography.**
   Tatiana Bubba, University of Ferrara, Italy.
2. **Perturbations of an elliptic PDE and applications to inverse problems and shape optimization.**
   Eric Bonnetier, Université Grenoble-Alpes, France.
3. **Introduction to the Boundary Control method.**
   Lauri Oksanen, University of Finland, Finland.
4. **Optimal Control and Machine Learning.**
   Donato Vásquez, Universidad de Chile, Chile.
5. **Microlocal analysis for the geodesic X-ray transform and boundary rigidity.**
   Andras Vasy, Stanford University, US.

Talks

1. **Some applications of the finite element method in inverse problems.**
   Jorge Aguayo, Universidad de Concepción, Chile.
2. **The inverse (poro)elasticity problem in biomedicine.**
   Nicolás Barnafi, Universidad Católica de Chile.
3. **Super-resolved multi-slice time-dependent deep image prior for cardiac cine MRI at 0.55T.**
   Evelyn Cueva, Universidad Católica de Chile.
4. **Simultaneous determination of wave speed, diffusivity and nonlinearity in the Westervelt equation.**
   Sebastián Acosta, Baylor College of Medicine & Texas Children's Hospital, US.
5. **TBD**
   Gunther Uhlmann, University of Washington, US, & Universidad Católica de Chile.

Abstracts of mini courses

Mathematics of X-ray Computed Tomography

Tatiana Bubba

University of Ferrara, Italy

In this course, I will give an introduction to theoretical and practical aspects of X-ray tomographic imaging. In particular, I will introduce the Radon transform and the filtered back-projection algorithm, and then focus on the challenges of image reconstruction from limited or noisy data, and how these can be faced using regularization theory. I will also touch upon the emerging field of tomographic reconstruction using deep learning techniques, covering topics such as learned priors, unrolled optimization algorithms, and end-to-end network architectures. Prior exposure to Fourier analysis, linear algebra, and basic functional analysis is recommended.

Perturbations of an elliptic PDE and applications to inverse problems and shape optimization

Eric Bonnetier

Université Grenoble-Alpes, France

Asymptotic expansions of the solution to an elliptic PDE in the presence of inclusions of small size have found successful applications in inverse problems, as a means to detect inhomogeneities from boundary measurements in a robust way.

In the course, we revisit some of this work and the applications it led to. We also consider perturbations on the boundary, such as replacing a Neumann boundary condition by a Dirichlet boundary condition (or vice-versa). We derive the general form of an asymptotic expansion between the solutions of the perturbed and unperturbed PDE's, and show that the convergence relies on an argument similar to compensated compactness.

We illustrate two of the applications of the resulting asymptotic formulas. We consider shape optimization problems, in which the subset of the boundary of a domain, where a specific boundary condition is applied, is part of the design variables. We also discuss the problem of how such asymptotic expansions can be used to achieve approximate cloaking.

Introduction to the Boundary Control method

Lauri Oksanen

University of Finland, Finland

The Boundary Control method can be used to solve several coefficient determination for wave equations. Furthermore, similar inverse problems for many other partial differential equations can be reduced to these problems. This is the case for inverse problems for heat and non-stationary Schrödinger equations, as well as for several fractional equations. Inverse problems for linear elliptic equations on a wave guide, and some non-linear elliptic equations are also covered by this theory. These lectures will give an introductory exposition of the method.

Optimal Control and Machine Learning

Donato Vásquez

Universidad de Chile, Chile

Machine Learning has achieved remarkable success in numerically solving complex problems. This progress has attracted considerable interest within the mathematical community, particularly in the application of machine learning techniques to compute optimal feedback laws. This task is known to suffer from the “curse of dimensionality”, namely, the exponential growth of the computational complexity with the dimensionality of the underlying dynamical system. This phenomenon is particularly important for the numerical resolution of control problems involving partial differential equations, as precise approximations require solving large finite-dimensional control problems.

In these lectures, we will focus on applications of machine learning techniques to the synthesis of optimal feedback laws and study how they can mitigate the curse of dimensionality. Both supervised and unsupervised techniques will be considered, and their theoretical and practical aspects will be compared. Furthermore, we will provide conditions which ensure the synthesis of approximate optimal feedback laws by these methods, supported by illustrative numerical experiments. This will allow participants to determine the potential and limitations of machine learning in the context of optimal control.

Microlocal analysis for the geodesic X-ray transform and boundary rigidity

András Vasy

Stanford University, US

The geodesic X-ray transform of a function on a manifold, possibly with boundary, such as a domain in an ambient Riemannian space, is the map associating to each geodesic within the manifold the integral of the function along the geodesic, defined under appropriate conditions, such as decay at infinity if the domain is non-compact. A basic question is the injectivity of this transform, together with estimates for the left inverse, and possible algorithms for constructing such a left inverse. There is also a tensorial version of this question; in this one takes a symmetric tensor and evaluates it on the tangent vector in each of its slots prior to integration, and one has to take into account the "obvious" nullspace: tensors which are the symmetric gradients of one lower rank tensors vanishing at the boundary/at infinity, called potential tensors. On the other hand, these problems are closely related to boundary rigidity: can one recover a Riemannian metric on a manifold with boundary from its distance function restricted to the boundary (in both slots)? The analogue of potential tensors here is the diffeomorphism invariance of the problem.

Dimensions three and higher have a different flavor from the two dimensional case; I will concentrate on three or higher dimensional manifolds. Much progress has been made on these problems in recent years, typically under some sort of convexity assumptions, such as the convexity of the boundary (for manifolds with boundary), and the existence of a potentially singular convex foliation. Analytically the heart of the approach is a geometric form of microlocal analysis, with the precise type adapted to the geometric setting.

In this minicourse I will explain the setting and the recent progress, in particular joint works with Gunther Uhlmann, Plamen Stefanov, Evangelie Zachos and Qiuye Jia. I will also explain the background analytic machinery, such as semiclassical foliation and scattering pseudodifferential operators, and how these are employed in these settings. I will in particular emphasize the role played by a smallness parameter, such as the semiclassical parameter in the first case and a domain thinness parameter in the second case, to deal with the a priori finite but potentially large dimensional nullspace that is typically given by microlocal tools.

Abstracts of Talks

Some applications of the finite element method to inverse problems

Jorge Aguayo

Universidad de Concepción, Chile

This talk will present some numerical results for solving elliptic optimal control problems using finite elements method and some applications in parameter identification problems. The theoretical results will be compared with numerical tests that aim to study the effect of subduction earthquakes and the detection of obstacles immersed in fluids.

The inverse (poro)elasticity problem in biomedicine

Nicolás Barnafi

Universidad Católica de Chile

The inverse elasticity problem can be simply stated as: given a deformed configuration and the forces that act on it, find an initial stress-free configuration such that when the given forces are applied to it, one recovers the given deformed configuration. Surprisingly, this problem can be framed as a (direct) elasticity one, whose mathematical properties are inherited from the original direct problem if the underlying material is sufficiently regular. In this talk, I will review this problem and its main mathematical properties. After this brief introduction, I will show some artifacts that appear when solving this problem, such as self-intersections and geometrically incompatible solutions. The talk will finish with an extension of this system to poroelastic materials, where I will show that the strong form of the equations does not allow for a weak formulation, and this requires some special treatment. All models will be shown to work in realistic heart geometries

Suoer-resolved multi-slice time-dependent Depp image prior for cardiac cine MRI at 0.55T

Evelyn Cueva

Universidad Católica de Chile

Magnetic Resonance Imaging (MRI) reconstruction can be formulated as an inverse problem involving undersampled Fourier measurements. In accelerated and low-field MRI, the problem becomes severely ill-posed due to aggressive k-space undersampling, low signal-to-noise ratio, and limited spatial resolution, particularly in dynamic cardiac imaging.

This talk presents a Deep Image Prior (DIP) approach for super-resolution in accelerated low-field MRI from an inverse problems perspective. DIP acts as an implicit regularizer by parametrizing the solution through a convolutional neural network optimized directly on the measured data, without external training. We discuss the role of architectural bias and early stopping as regularization mechanisms, and introduce a multi-scale formulation that couples low- and high-resolution reconstructions. Structured latent-space modeling is used to exploit temporal and spatial redundancies, enabling coherent dynamic super-resolution in challenging MRI settings

Simultaneous determination of wave speed, diffusivity and nonlinearity in the Westervelt equation

Sebastian Acosta

Baylor College of Medicine and Texas Children's Hospital, US

The Westervelt equation models the propagation of high-intensity ultrasound waves. For medical imaging purposes, it is needed to estimate the parameters in the equation that model wave speed, diffusivity, and nonlinearity. We show that, by constructing time-periodic solutions excited from the boundary time-harmonically at a sufficiently high frequency, knowledge of the first- and second-harmonic Cauchy data at the boundary is sufficient to simultaneously determine the wave speed, diffusivity and nonlinearity in the interior of the domain of interest.

TBD

Gunther Uhlmann

University of Washington, US, & Universidad Católica de Chile

TBD