

Mostly Maximum Principle

5th edition: in Latin America for the first time

Book of Abstracts

Claudemir Alcantara

Pontifícia Universidade Católica do Rio de Janeiro, Brazil

Title: Hessian regularity in H^k older spaces for a semi-linear bi-Laplacian equation.

Abstract: We examine a semi-linear variant of the bi-Laplacian equation in the superlinear, subquadratic setting and obtain C^2 -regularity estimates, depending on the growth regime of the nonlinearity. Our strategy is to render this fourth-order problem as a system of two Poisson equations and explore the interplay between the integrability and smoothness available for each equation taken isolated. We conclude the talk with a discussion on further directions of research.

This is joint work with Edgard A. Pimentel and José Miguel Urbano.

Damião J. Araújo

Universidade Federal da Paraíba, Brazil

Title: Fully nonlinear dead-core systems

Abstract: In this talk, we explore fully nonlinear dead-core systems coupled with strong absorption terms. Our investigation reveals a chain reaction mechanism, utilizing the properties of an equation within the system to achieve higher sharp regularity across the free boundary. Furthermore, we establish geometric measure estimates and derive coincidence property for the free boundaries. We also obtain Liouville-type results for entire solutions. Notably, these findings are novel even when applied to linear systems.

Julian Bonder

Universidad de Buenos Aires, Argentina

Title: The eigenvalue problem for the fractional generalized Laplacian

Abstract: In this talk, I will discuss the eigenvalue problem for the fractional generalized Laplacian. This problem is nonlinear and nonhomogeneous, presenting challenges in its analysis and solution.

Maissâ Boughrara

Université Sorbonne Paris Nord, France

Title: Blow-up profile for a nonlinear heat equation with a gradient term

Abstract: We consider a nonlinear heat equation with a gradient term . In the earlier literature, we have the existence of a blow-up solution with an explicit profile. By pursuing a limited development, we are trapped in scales of order $\frac{1}{\log|T-t|^\nu}$ where T is the blow-up time. In this work, we are able to provide a sharper description up to $|T-t|^\mu$ with a new profile.

Juan Pablo Cabeza

Universidad de Chile, Chile

Title: Holder Regularity Theory for Nonlocal Operators

Abstract: In this talk we discuss some regularity results for viscosity solutions of nonlocal uniformly elliptic second order differential equations of gradient terms and fully nonlinear type.

Loth Damagui Chabi

Université Sorbonne Paris Nord, France

Title: Refined behavior and structural universality of the blow-up profile for the semilinear heat equation with non scale invariant nonlinearity

Abstract: We consider the semilinear heat equation

$$u_t - \Delta u = f(u)$$

for a large class of non scale invariant nonlinearities of the form $f(u) = u^p L(u)$, where $p > 1$ is Sobolev subcritical and L is a slowly varying function (which includes for instance logarithms and their powers and iterates, as well as some strongly oscillating functions). For any positive radial decreasing blow-up solution, we obtain the sharp, global blow-up profile in the scale of the original variables (x, t) , which takes the form:

$$u(x, t) = (1 + o(1)) G^{-1} \left(T - t + \frac{p-1}{8p} \frac{|x|^2}{|\log|x||} \right), \text{ as } (x, t) \rightarrow (0, T),$$

where $G(X) = \int_X^\infty \frac{ds}{f(s)}$. This estimate in particular provides the sharp final space profile and the refined space-time profile. As a remarkable fact and completely new observation, our results reveal a *structural universality* of the global blow-up profile, being given by the “resolvent” G^{-1} of the ODE, composed with a universal, time-space building block, which is the same as in the pure power case.

Joint work with Ph. Souplet.

Rayssa Caju

Universidad de Chile, Chile

Title: Large conformal metrics with prescribed Gaussian and geodesic curvatures

Abstract: In this talk, our goal is to discuss the existence of at least two distinct conformal metrics with prescribed Gaussian curvature $K_g = f + \lambda$ and geodesic curvature $k_g = h + \mu$, where f and h are nonpositive functions, and λ and μ are positive constants. Utilizing Struwe's monotonicity trick, we investigate the blowup behavior of the solutions and establish a non-existence result for the limiting PDE, eliminating one of the potential blow-up profiles.

This work is joint with Tiarlos Cruz and Almir Silva Santos.

João Vitor da Silva

Universidade Estadual de Campinas, Brazil

Title: Sharp regularity for the obstacle problem for quasilinear elliptic equations

Abstract: This lecture focuses on sharp gradient estimates for weak solutions to an obstacle problem involving a quasilinear operator with unbounded source terms. Specifically, the speaker addresses an explicit and universal regularity exponent (dependent only on universal parameters) for the gradient of the solutions. These findings are significant even for the simplest model case governed by the p -Laplacian with Hölder continuous coefficients.

The work was developed in collaboration with Elzon C. Bezerra Júnior (UFCA, Brazil) and Romário T. Frias (UNICAMP, Brazil).

Juliana Fernandes

Universidade Federal do Rio de Janeiro, Brazil

Title: The Nehari manifold for a degenerate logistic parabolic equation

Abstract: Our main goal is to exploit the interplay between variational methods and dynamical systems. In particular, we use the so-called Nehari manifold to provide a picture of the phase plane, divided into regions of convergence over time. For the related semilinear stationary elliptic equation, general questions on the existence, multiplicity, and nonexistence of positive solutions have been discussed. The existence problem for sign-changing solutions is somewhat more delicate, with only a few results available in the literature. We tackle this problem by using Ambrosetti and Rabinowitz Mountain Pass Theorem on Nehari.

This is a joint work with L. Maia.

Pedro Gaspar

Pontificia Universidad Catolica de Chile, Chile

Title: Heteroclinic solutions of the Allen-Cahn equation and mean curvature flows

Abstract: The Allen-Cahn equation is a semilinear partial differential equation which models phase transition and separation phenomena and which provides a regularization for the mean curvature flow (MCF), one of the most studied geometric flows.

In this talk, we combine analytic, geometric and topological strategies to obtain existence results for eternal solutions of this parabolic PDE connecting unstable nontrivial stationary solutions, namely heteroclinic solutions, in certain compact manifolds. In the concrete setting of a 3-dimensional round sphere, we describe the space of all low energy eternal solutions and explain how they can be used to construct geometrically interesting MCFs.

This is joint work with Jingwen Chen (University of Pennsylvania).

Nikola Kamburov

Pontificia Universidad Católica de Chile, Chile

Title: One-phase free boundary solutions of finite index

Abstract: The classical one-phase free boundary problem (FBP) is one of the prototypical free boundary problems. Starting from the pioneering work of Alt and Caffarelli (1981), its energy-minimizing solutions have been fairly well studied and understood. The focus of the work that I will present in this talk is on solutions of the one-phase FBP that are not necessarily energy minimizing. In joint work with J. Basulto (PUC-Chile), we investigated entire solutions of bounded Morse index and obtained a complete classification theorem in the plane as well as a partial rigidity result for stable solutions in Euclidean 3-space. Our results are free boundary counterparts to classical theorems in the minimal surface literature.

Shuheii Kitano

Waseda University, Japan

Title: Calderón-Zygmund estimates for fully nonlinear local and nonlocal equations

Abstract: In my talk, we will discuss about Calderón-Zygmund estimates for both fully nonlinear second-order and nonlocal equations. In the first half of the talk, I will present the Calderón-Zygmund estimate for fully nonlinear nonlocal equations, which is analogue of those for second-order equations by Caffarelli. In the second half of the talk, I will present the Calderón-Zygmund estimate with the exponent $p=1$ for fully nonlinear second-order equations by our new approach.

Shigeaki Koike

Waseda University, Japan

Title: A PDE approach to a price formation model

Abstract: We consider a system of Hamilton-Jacobi and Fokker-Planck equations and an integral equation, which describe a balance of demand and supply of "energy" for instance. In 2021, Gomes and Saude studied this system as a constrained mean-field-game. We discuss this system with pure PDE arguments.

This is a joint work with S. Ishibashi, T. Kosugi.

Fabiana Leoni

Sapienza Università di Roma, Italy

Title: Fully nonlinear equations with singular terms in punctured balls

Abstract: We consider radial solutions of fully nonlinear, uniformly elliptic equations, posed in punctured balls, in presence of radial singular quadratic potentials. We discuss both the principal eigenvalues problem, obtaining an extension in the fully nonlinear framework of the Hardy-Sobolev constant, and the classification of solutions based on the asymptotic behavior near the singularity, for equations having also superlinear zero order terms.

The results are based on joint works with I. Birindelli and F. Demengel.

Liliane Maia

Universidade de Brasília, Brazil

Title: An upper bound for the least energy of a sign-changing solution to a zero mass problem

Abstract: In this talk we present some recent results on an upper bound for the least energy of a sign-changing solution to the the nonlinear scalar field equation

$$-\Delta u = f(u), \quad u \in D^{1,2}(R^N),$$

where $N \geq 5$ and the nonlinearity f is subcritical at infinity and supercritical near the origin. More precisely, we establish the existence of a nonradial sign-changing solution whose energy is smaller than $12c_0$ if $N = 5, 6$ and smaller than $10c_0$ if $N \geq 7$, where c_0 is the ground state energy.

Work in collaboration with Mónica Clapp and Benedetta Pellacci.

Javier Monreal

Universidad de Chile, Chile

Title: A singular perturbation problem modeling amplitude walls of striped patterns

Abstract: In this talk, I will present some results of a singular perturbation problem. I will discuss existence results both when the perturbation parameter is considered and when it is zero, demonstrating their continuous connection as the parameter approaches zero. Finally, I will show that the behavior of one of the unknowns can be modeled using a Painlevé II equation in the limit.

Diego Moreira

Universidade Federal do Ceará

Title: Flipping one sided regularity via a Harnack approach and applications to nonlinear elliptic problems

Abstract: In this talk we discuss some recent advances on the regularity theory of non linear elliptic problems showing that weak Harnack type arguments allow the

passage from one-side regularity to full regularity in Hölder and Sobolev spaces. As a particular case of these phenomena, we can identify the Caffarelli, Kohn, Nirenberg and Spruck theorem (in the 80s), as well as, some more recent regularity results obtained together with Alessio Figalli (ETH) and Ederson Braga (UFC) both on the regularity of semiconvex supersolutions of uniformly elliptic equations. This problem has some motivations linked to free boundary problems as well.

This is a joint work with Edgard Pimentel (University of Coimbra).

Filomena Pacella

Sapienza Università di Roma, Italy

Title: One-dimensional solutions in cylinders : energy analysis and bifurcation

Abstract: We consider semilinear elliptic problems in cylindrical domains in which 1-dimensional solutions (namely solutions which depend only on the variable which describes the axis of the cylinder) represent a special class of solutions. We analyze some features of these solutions, in particular with respect to energy optimization and a related overdetermined problem. We present several results which show that these solutions and the simple domains where they are defined (namely bounded cylinders) are not always the best candidates to minimize the energy. This, in turn, suggests the existence of nontrivial domains for which the overdetermined problem admits a solution which can be obtained by a bifurcation analysis. The proof of these results are based on a careful study of the second derivative of the energy functional and/or on the analysis of the linearization of a normal derivative operator and show that a crucial role is played by the first Neumann eigenvalue of the Laplace operator on the domain which spans the cylinder.

The talk is based on some joint works with D.G.Afonso, A.Iacopetti, D.Ruiz, P.Sicbaldi.

Yehuda Pinchover

Technion-Israel Institute of Technology, Israel

Title: The Landis conjecture via Liouville comparison principle and criticality theory

Abstract: We present partial affirmative answers to Landis conjecture in all dimensions for two different types of linear, second order, elliptic operators in a domain $\Omega \subset R^N$. In particular, we give a sharp criterion when a solution of a Schrödinger equation in R^N with a potential $V \leq 1$ is trivial. Moreover, we address the analogue of Landis conjecture for quasilinear problems.

Alexander Quaas

Universidad Santa María, Chile

Title: Large-time behavior of unbounded solutions of viscous Hamilton-Jacobi equations in R^n

Abstract: We study the large-time behavior of bounded from below solutions of parabolic viscous Hamilton-Jacobi Equations in the whole space.

Jean-Michel Roquejoffre

Université Toulouse III-Paul Sabatier, France

Title: A reaction-diffusion system of the thermo-diffusive type: boundedness of the solutions

Abstract: The seemingly innocent question addressed in this talk is the global in time boundedness of the positive solutions of a reaction-diffusion system, with different diffusions for each component, and a polynomial nonlinearity that satisfies a balance law. When posed in a bounded domain, this class of system has, for a long time, been known to admit solutions that are bounded in time. As opposed to that, there is no definite answer when the domain is the whole space. Solutions have been shown to grow at most logarithmically in time, and the existing boundedness proofs are not complete. The goal of the talk is to discuss the boundedness when the nonlinearity, as well as having some polynomial bounds, satisfies an additional nondegeneracy assumption.

Joint work with J. La and L. Ryzhik.

Julio Daniel Rossi

Universidad de Buenos Aires, Argentina

Title: Convex functions

Abstract: We introduce a notion of convexity with respect to a one-dimensional operator that extends naturally the usual notion of convexity in the Euclidean space. With this notion of convexity, we study the convex envelope inside a domain of a boundary datum (the largest possible convex function inside the domain that is below the datum on the boundary) and show that the convex envelope is characterized as a viscosity solution to a PDE that is given by the infimum among all possible directions of the 1-dimensional operator. For this equation we prove existence, uniqueness and a comparison principle (in the framework of viscosity solutions). In addition, we study the behavior of the convex envelope when a parameter that appears in the 1-dimensional operator goes to zero or to one, recovering the classical convex envelope or the quasiconvex envelope.

Joint work with P. Blanc and M. Parviainen.

Pieralberto Sicbaldi

Universidad de Granada, Spain

Title: A Schiffer-type problem in annuli and applications to Euler flows

Abstract: If on a smooth bounded domain of the plane there is a (nonconstant) Neumann Laplace eigenfunction that is locally constant on the boundary, must the domain be a disk or an annulus? This question can be understood as a weaker analog of the well known Schiffer conjecture. In fact, here the eigenfunction is allowed to take a different constant value on each connected component of the boundary. Many of the known rigidity properties of the original Schiffer problem are essentially preserved.

In this talk I will show that the answer to such question is negative by constructing a family of nontrivial doubly connected domains with the above property. Furthermore, I will show that this implies the existence of continuous, compactly supported stationary

weak solutions to the 2D incompressible Euler equations which are not locally radial. This talk is based on a joint work with A. Enciso, A. J. Fernández and D. Ruiz.

Antonio Siconolfi

Sapienza Università di Roma, Italy

Title: Time-dependent Weak KAM theory

Abstract: We adapt the framework of Weak KAM theory for periodic Hamiltonian to the time-dependent case through tools from optimal transport. In this context we give in particular the notion of Aubry and Mather trajectories. The static Mather measures play the role of equilibria of a suitable dynamic defined in the space of measures. We further provide a proof of a Brenier-Benamou type formula adapted to this setting. The Hamiltonians we consider satisfy rather general assumptions, and are not Tonelli.

Mayra Soares Costa Rodrigues

Universidade de Brasília, Brazil

Title: Boundary Weak Harnack Estimates and Regularity for Elliptic Operators in Divergence Form and Applications in PDEs

Abstract: We obtain a global extension of the classical Weak Harnack Inequality, which extends and quantifies the Hopf-Oleinik boundary-point lemma, for uniformly elliptic equations in divergence form, under the weakest assumptions on the leading coefficients and on the boundary of the domain. Our main tool is the use of suitable barrier functions, which are solutions of auxiliary problems and the C^1 -estimates up to the boundary. Among the consequences is a boundary gradient estimate, due to Krylov and well-studied for non-divergence form equations, but completely novel in the divergence framework. Another consequence is a new, more general version of the Hopf-Oleinik lemma. Furthermore, we provide an application showing how to use these results in order to deduce a priori upper bounds and multiplicity of solutions for a class of quasilinear elliptic problems with quadratic growth on the gradient.

Aelson Sobral

Universidade Federal da Paraíba, Brazil

Title: Maximum principles for parabolic equations of infinity Laplacian type

Abstract: In this presentation, we explore the validity of ABP-type maximum principles applied to viscosity solutions of parabolic equations involving the infinity-Laplacian operator. We leverage these estimates to derive stable-in- p , up to the boundary, continuity estimates for solutions to the parabolic normalized p -Laplacian equation and their limit as p tends to infinity.

Andrzej Swiech

Georgia Institute of Technology, United States

Title: ABP maximum principle for L^p -viscosity solutions of parabolic equations with singular terms

Abstract: We will present several results on Aleksandrov-Bakelman-Pucci and Bony type maximum principles for L^p -viscosity solutions of fully nonlinear, uniformly parabolic equations with singular drift terms. In particular, we will discuss how to obtain the ABP maximum principle in a version with contact sets for viscosity solutions of such equations and what integrability of the gradient coefficient is required. Implications of the new maximum principle results for the theory of L^p -viscosity solutions will also be discussed.

This is a joint work with S. Koike.

Hugo Tavares

Instituto Superior Técnico, Universidade de Lisboa, Portugal

Title: Spectral partition problems with volume and inclusion constraints: existence and regularity results

Abstract: In this talk we discuss a class of spectral partition problems with a measure constraint, for partitions of a given bounded connected open set. We establish the existence of an optimal open partition, showing that the corresponding eigenfunctions are locally Lipschitz continuous, and obtain some qualitative properties for the partition. Then, we prove a full regularity result for the associate free boundary, for a particular solution. The proofs use an equivalent weak formulation that involves a minimization problem of a penalized functional where the variables are functions rather than domains, suitable deformations, blowup techniques and monotonicity formulas.

The talk is based in two works, one joint with Ederson Moreira dos Santos (ICMC-USP), Pêdra Andrade and Makson Santos (IST-Lisboa), the other in collaboration with Dario Mazzoleni (Pavia) and Makson Santos.

Erwin Topp

Universidade Federal do Rio de Janeiro, Brazil

Title: Nonlocal Hamilton-Jacobi equation on networks

Abstract: A network is a set made by a finite number of vertices connected through edges. Due to its application on traffic flow models on graphs, differential equations posed on networks have attracted the attention of the mathematical community in the last two decades. In this talk, I will report recent results about the existence, uniqueness, and regularity of (viscosity) solutions for nonlocal Hamilton-Jacobi equations of stationary type, posed on networks. The intriguing role of the vertices (which is neither an interior nor a boundary point, or perhaps both!), together with its interaction with the nonlocality of the operators, create nontrivial difficulties in the implementation of the available techniques. Joint work with Olivier Ley (INSA-Rennes, France).