Satisfying Instead of Optimising in the Nash Demand Game

Sigifredo Laengle[1]

Faculty of Economics and Business, University of Chile

Online, July 22, 2020

[1]The author sincerely thanks the invitation of F. Flores-Bazán and A. Hantoute to the *Optimization Seminars* organised by UCh and UdC.

Outline

The Nash Demand Game I I

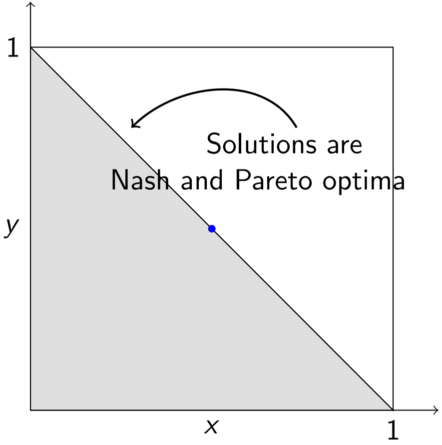

Figure: The classic Nash Demand Game. How to reach an equitable solution?

The Nash Demand Game II

We will start with a short description of the Nash Demand Game (Nash 1953):

The Nash Demand Game II

We will start with a short description of the Nash Demand Game (Nash 1953):

1. Let be a *pie* of size 1 to be distributed among two negotiators: Emil and Frances.

The Nash Demand Game II

We will start with a short description of the Nash Demand Game (Nash 1953):

1. Let be a *pie* of size 1 to be distributed among two negotiators: Emil and Frances.
2. They choose simultaneously strategies x and y respectively.

The Nash Demand Game II

We will start with a short description of the Nash Demand Game (Nash 1953):

1. Let be a *pie* of size 1 to be distributed among two negotiators: Emil and Frances.
2. They choose simultaneously strategies x and y respectively.
3. They try to maximise the utility function given by $f_E(x, y) \doteq x$ and $f_F(x, y) \doteq y$ for Emil and Frances respectively.

The Nash Demand Game II

We will start with a short description of the Nash Demand Game (Nash 1953):

1. Let be a *pie* of size 1 to be distributed among two negotiators: Emil and Frances.
2. They choose simultaneously strategies x and y respectively.
3. They try to maximise the utility function given by $f_E(x, y) \doteq x$ and $f_F(x, y) \doteq y$ for Emil and Frances respectively.
4. The strategy pair $(\overline{x}, \overline{y})$ is a *Nash equilibrium* of the game iff

$$f_E(\overline{x}, \overline{y}) = \max_x \{f_E(x, \overline{y}) : x \geq 0, \text{ and } x + \overline{y} \leq 1\};$$

and

$$f_F(\overline{x}, \overline{y}) = \max_y \{f_F(\overline{x}, y) : y \geq 0, \text{ and } \overline{x} + y \leq 1, \}.$$

Some Open Issues

Two important issues in distributive bargaining theory are:

1. In case of agreement, it is not possible to discriminate between multiple equilibria.

Some Open Issues

Two important issues in distributive bargaining theory are:

1. In case of agreement, it is not possible to discriminate between multiple equilibria.
2. A *negotiation breakdown* is not possible, despite of the degree of polarisation is too *high*.

Some Open Issues

Two important issues in distributive bargaining theory are:

1. In case of agreement, it is not possible to discriminate between multiple equilibria.
2. A *negotiation breakdown* is not possible, despite of the degree of polarisation is too *high*.
3. Important questions about equitable solutions are present.

Subsequent literature has explored both issues, but in general through an high sophistication of the original game:

1. Introducing perturbations into the original game.

Some Open Issues

Two important issues in distributive bargaining theory are:

1. In case of agreement, it is not possible to discriminate between multiple equilibria.
2. A *negotiation breakdown* is not possible, despite of the degree of polarisation is too *high*.
3. Important questions about equitable solutions are present.

Subsequent literature has explored both issues, but in general through an high sophistication of the original game:

1. Introducing perturbations into the original game.
2. Adopting an incomplete information environment or a dynamic approach.

Some Open Issues

Two important issues in distributive bargaining theory are:

1. In case of agreement, it is not possible to discriminate between multiple equilibria.
2. A *negotiation breakdown* is not possible, despite of the degree of polarisation is too *high*.
3. Important questions about equitable solutions are present.

Subsequent literature has explored both issues, but in general through an high sophistication of the original game:

1. Introducing perturbations into the original game.
2. Adopting an incomplete information environment or a dynamic approach.
3. Or the proposition of a new game at all.

Open Issues Overview

There are a enormous research effort for understanding the behaviour of negotiator or parties:

1. Axiomatic formulations inspired by Nash's own work.
    - Nash Bargaining Solution (Nash 1953).
    - Kalai–Smorodinsky bargaining solution (Kalai and Smorodinsky 1975).
    - Egalitarian bargaining solution (Kalai 1977).

Open Issues Overview

There are a enormous research effort for understanding the behaviour of negotiator or parties:

1. Axiomatic formulations inspired by Nash's own work.
   - Nash Bargaining Solution (Nash 1953).
   - Kalai–Smorodinsky bargaining solution (Kalai and Smorodinsky 1975).
   - Egalitarian bargaining solution (Kalai 1977).
2. Generalisations, amplifications, and new considerations of the basic model.
   - A compendium of the bargaining problem in (Muthoo 1999).
   - Considerations as an alternating-offer bargaining, risk of breakdown, outside and inside options, asymmetric information, risk aversion, impatient, and envy.

Open Issues Overview

There are a enormous research effort for understanding the behaviour of negotiator or parties:

1. Axiomatic formulations inspired by Nash's own work.
   - Nash Bargaining Solution (Nash 1953).
   - Kalai–Smorodinsky bargaining solution (Kalai and Smorodinsky 1975).
   - Egalitarian bargaining solution (Kalai 1977).
2. Generalisations, amplifications, and new considerations of the basic model.
   - A compendium of the bargaining problem in (Muthoo 1999).
   - Considerations as an alternating-offer bargaining, risk of breakdown, outside and inside options, asymmetric information, risk aversion, impatient, and envy.
   - All extension closely related to a normative specification of the loss functions.
3. Experimental studies found no consistent support for any of the bargaining models (Schellenberg 1990).

Trajectories Movements

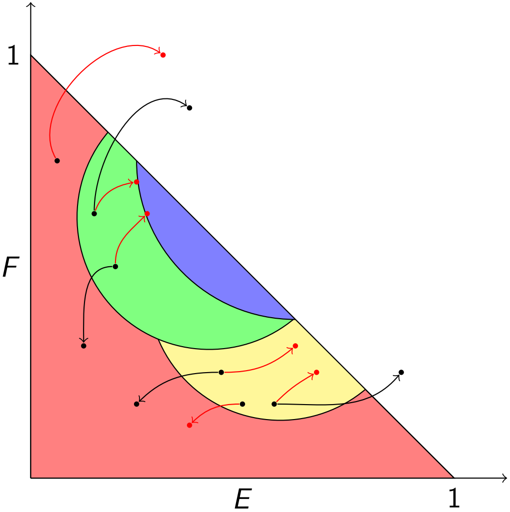

Figure: Trajectories describe the bargaining behaviour between different subsets.

Some Basic Characteristics

Some basic characteristics of the trajectories behaviour:

1. Can be described by movements between subsets of the set of possible agreements.

Some Basic Characteristics

Some basic characteristics of the trajectories behaviour:

1. Can be described by movements between subsets of the set of possible agreements.
2. If negotiators have **necessity** of an agreement, they need **chances** for driving the bargaining trajectories.

Some Basic Characteristics

Some basic characteristics of the trajectories behaviour:

1. Can be described by movements between subsets of the set of possible agreements.
2. If negotiators have **necessity** of an agreement, they need **chances** for driving the bargaining trajectories.
3. It is a matter of *satisfying* instead of *optimising*.

Some Basic Characteristics

Some basic characteristics of the trajectories behaviour:

1. Can be described by movements between subsets of the set of possible agreements.
2. If negotiators have **necessity** of an agreement, they need **chances** for driving the bargaining trajectories.
3. It is a matter of *satisfying* instead of *optimising*.

Definitions I

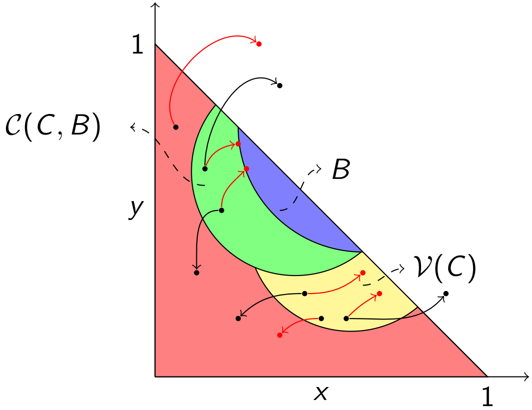

Figure: $\mathcal{C}(C,B) \doteq \mathcal{V}(C) \cup \mathcal{V}(C,B)$ is added. Four subsets are defined.

Definitions I

For modelling the classic Nash Demand Game we propose the following ingredients:

1. The **event space**, a real Hilbert space $\mathcal{H}$.
2. $\emptyset \neq C \subset \mathcal{H}$ the **bargaining set**. It is compact and convex. A subset of C is the **possible agreement set**.
3. (x, y) elements of C are **states of negotiation** or demands for the pie. In some cases are agreements, solutions, and optima.
4. $(u, v) \in U \times V$ are **decisions** (or bistrategies) of Emil and Frances respectively. $U \times V$ is a nonempty compact convex subset of $\mathcal{H}$.
5. $\varphi : C \to 2^{\mathcal{H}}$ the **bargaining rule**. It is a upper hemicontinuous set-valued operator with nonempty closed and convex values.
6. The **equilibria set** of the bargaining rule φ is the set of zeros of φ, i.e. $\operatorname{zer} \varphi \doteq \{(x, y) \in C : 0 \in \varphi(x, y)\}$.

Definitions II

We will add the following ingredients:

1. The bargaining behaviour can be described by a **bargaining trajectory** $\{(x_n, y_n)\}_{n\in\mathbb{N}}$, a sequence contained in $\mathcal{H}$.
2. A bargaining trajectory is a **viable trajectory** if $\{(x_n, y_n)\}_{n\in\mathbb{N}} \subset C$.
3. Given an initial state of negotiation $(x, y) \in \mathcal{H}$, let $\{(x_n, y_n)\}_{n\in\mathbb{N}}$ be a **trajectory driven** by the the bargaining rule φ if $(x_0, y_0) \doteq (x, y)$ and

$$(\forall n \in \mathbb{N})\ (x_{n+1}, y_{n+1}) - (x_n, y_n) \in \varphi(x_{n+1}, y_{n+1}).$$

Definitions III

4. We define $\mathcal{V}(C)$ the **viability kernel of** C as the set of initial states $(x, y) \in C$ such that there exists a viable trajectory such that
   - $(x_0, y_0) \doteq (x, y)$ and
   - $(\forall n \in \mathbb{N})\ (x_n, y_n) \in C$.

   .

5. Let $B \subset C$ a **target set**. Let $\mathcal{C}(C, B)$ the **viable capture basin** of C with target B is the set of initial states $(x, y) \in C$ such that there exists a trajectory such that
   - $(x_0, y_0) \doteq (x, y)$,
   - $(\forall n \in \{0, \ldots, k\})\ (x_n, y_n) \in C$, and
   - $(x_k, y_k) \in B$.

   .

6. Let $\mathcal{V}(C, B)$ be the **viability kernel with target** C given by

$$\mathcal{V}(C, B) \doteq \mathcal{V}(C) \cup \mathcal{C}(C, B).$$

Main Theorem I

These behavioural archetypes can be addressed with the Viability Theory (Aubin 2009; Aubin, Bayen, and Saint-Pierre 2011; Aubin 2013). Let $\mathcal{H}$ be a Hilbert Space, $T_C : \mathcal{H} \to 2^{\mathcal{H}}$ the tangent cone to C at x.

Theorem (Existence of Equilibrium)

Let $\emptyset \neq C \subset \mathcal{H}$ be the compact convex bargaining set. Let $\varphi : C \to 2^{\mathcal{H}}$ a bargaining rule that is an upper-hemicontinuous set-valued operator with nonempty closed convex values. If C satisfies the tangential condition

$$(\forall x \in C)\ T_C x \cap \varphi x \neq \emptyset,$$

then the following hold

(i) $\operatorname{zer} \varphi \neq \emptyset$ *and*

(ii) $(\forall y \in C)(\exists x \in C)\ y \in x - \varphi x.$

Modelling Minimal Power I

Let be the following components of the bargaining problem:

1. the event space $\mathcal{H} \doteq \mathbb{R}^2$,
2. the set of possible agreements

$$C \doteq \{(x, y) \in \mathbb{R}^2 : x, y \in \mathbb{R}_+ \text{ and } x + y \leq 1\}, \text{ and}$$

3. the *linear* bargaining rule given by

$$\varphi : C \to 2^{\mathcal{H}} : (x, y) \mapsto \varphi(x, y) \doteq (\alpha x + \beta y + u, \beta x + \alpha y + v),$$

where the decisions $u, v \in [-\mu, +\mu]$ (i.e. $U = V \doteq [-\mu, +\mu]$), $\mu \in \mathbb{R}_+$ and $\alpha, \beta \in \mathbb{R}$.

Proposition

Let the bargaining problem defined as above, then if $\mu \geq \max\{|\alpha|, |\beta|, \alpha + \beta\}$ *the following hold* $\mathcal{V}(C) = C$.

Modelling Minimal Power II

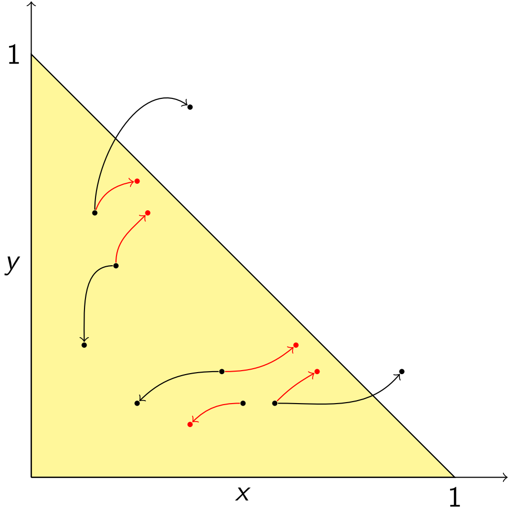

Figure: If $\mu \geq \max\{|\alpha|, |\beta|, \alpha + \beta\}$ the following hold $B \doteq C$ and $\mathcal{C}(C, B) = C$.

The Nash Bargaining Solution I

Let be $\mathcal{H}$ and C as before. The negotiators cooperate and agree to have the same loss function $f : \mathcal{H} \to]-\infty, +\infty] : (x, y) \mapsto -xy + \iota_C(x, y)$, thus the *linear* bargaining rule will be

$$\varphi : C \to 2^{\mathcal{H}} : (x, y) \mapsto \varphi(x, y) \doteq (-y, -x) - N_C(x, y),$$

where $N_C : \mathcal{H} \to 2^{\mathcal{H}}$ is the normal cone to C at $x \in C$ and ι_C is the indicator function of the set C.
In this case the target set is $B \doteq \{(\frac{1}{2}, \frac{1}{2})\}$, we have the following proposition:

Proposition

Let the bargaining problem defined as before, then

$$\mathcal{V}(C, B) = \mathcal{V}(C) = C = \mathcal{C}(C, B).$$

The Nash Bargaining Solution II

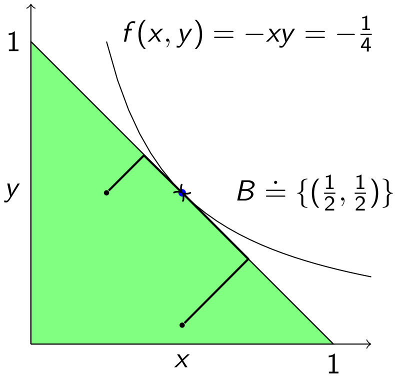

Figure: Every possible agreement arrives to B.

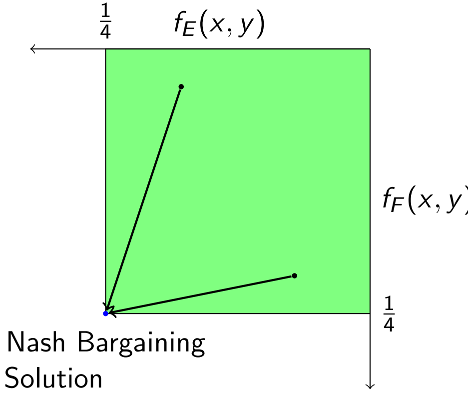

Figure: Trajectories of loss functions.

A Polarised Nash Demand Game I

The Polarised Nash Demand Game is an extension of the classical Nash Demand Game (Laengle and Loyola 2012). It considers that negotiators do not cooperate and they have envy. They have the following loss functions:

$$f_E(x, y) : \mathcal{H} \to]-\infty, +\infty] : (x, y) \mapsto -x + \lambda y + \iota_C(x, y),$$

and

$$f_F(x, y) : \mathcal{H} \to]-\infty, +\infty] : (x, y) \mapsto -y + \lambda x + \iota_C(x, y).$$

where $\lambda \in [0, 1]$ is an *envy factor*. Thus the *linear* bargaining rule will be

$$\varphi : C \to 2^{\mathcal{H}} : (x, y) \mapsto \varphi(x, y) \doteq (1 - (x + y), 1 - (x + y)) - N_C(x, y).$$

A Polarised Nash Demand Game II

Proposition

Let the bargaining problem defined as before, then the target set is given by

$$B = \{(x, y) \in C : x, y \in [\lambda(1+\lambda)^{-1}, (1+\lambda)^{-1}] \text{ and } x + y = 1\},$$

then

$$\mathcal{V}(C, B) =, \text{ and}$$

$$\mathcal{V}(C, B) = \mathcal{V}(C) = \mathcal{C}(C, B) \subset C.$$

A Polarised Nash Demand Game III

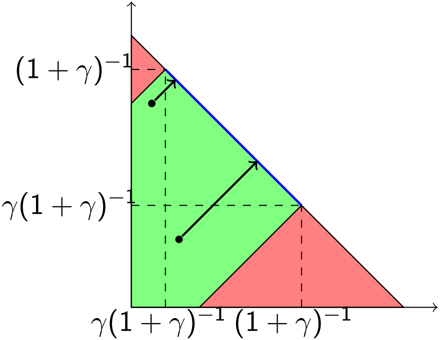

Figure: Not every possible agreement arrives to B.

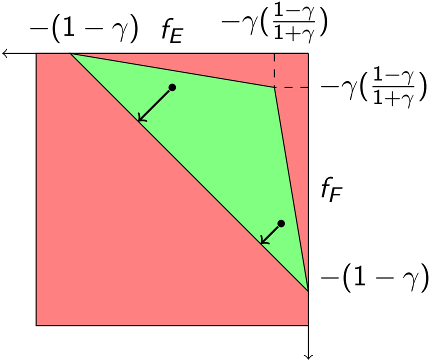

Figure: Trajectories of loss functions.

Conclusions

The main conclusions of this work are:

1. New strategic behaviour archetypes:
   - Can be described by movements between subsets of the set of possible agreements.

Conclusions

The main conclusions of this work are:

1. New strategic behaviour archetypes:
    - Can be described by movements between subsets of the set of possible agreements.
    - Chance and necessity as behaviour principles.

Conclusions

The main conclusions of this work are:

1. New strategic behaviour archetypes:
   - Can be described by movements between subsets of the set of possible agreements.
   - Chance and necessity as behaviour principles.
   - The classic models derived from the Nash Demand Game are models that are restricted to a few patterns of behaviour.

Conclusions

The main conclusions of this work are:

1. New strategic behaviour archetypes:
   - Can be described by movements between subsets of the set of possible agreements.
   - Chance and necessity as behaviour principles.
   - The classic models derived from the Nash Demand Game are models that are restricted to a few patterns of behaviour.
2. Perspectives:
   - The basic Nash Demand Game model has been extended in multiple dimensions.

Conclusions

The main conclusions of this work are:

1. New strategic behaviour archetypes:
    - Can be described by movements between subsets of the set of possible agreements.
    - Chance and necessity as behaviour principles.
    - The classic models derived from the Nash Demand Game are models that are restricted to a few patterns of behaviour.
2. Perspectives:
    - The basic Nash Demand Game model has been extended in multiple dimensions.
    - The framework presented here would allow such dimensions to be articulated in a single theory.

References I

- Aubin, Jean-Pierre (2009). *Viability theory*. Springer Science & Business Media.
- – (2013). *Optima and equilibria: an introduction to nonlinear analysis*. Vol. 140. Springer Science & Business Media.
- Aubin, Jean-Pierre, Alexandre M Bayen, and Patrick Saint-Pierre (2011). *Viability theory: new directions*. Springer Science & Business Media.
- Kalai, Ehud (1977). "Proportional solutions to bargaining situations: interpersonal utility comparisons". In: *Econometrica: Journal of the Econometric Society*, pp. 1623–1630.
- Kalai, Ehud and Meir Smorodinsky (1975). "Other Solutions to Nash's Bargaining Problem". In: *Econometrica* 43.3, pp. 513–518.
- Laengle, Sigifredo and Gino Loyola (2012). "Bargaining and negative externalities". In: *Optimization Letters* 6.3, pp. 421–430.
- Muthoo, Abhinay (1999). *Bargaining theory with applications*. Cambridge University Press.

References II

Nash, John (1953). "Two-person cooperative games". In: *Econometrica: Journal of the Econometric Society*, pp. 128–140.

Schellenberg, James A (1990). "'Solving' the Bargaining Problem". In: *Mid-American Review of Sociology*, pp. 77–88.