Groundstates of critical and supercritical problems of Brezis-Nirenberg type

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We study the existence of rotationally-invariant ground states to supercritical problems of the form

$$-\Delta v = \lambda v + |v|^{p-2}\, v \;\text{ in } \Omega, \qquad v = 0 \text{ on } \partial\Omega,$$

in an $O(k+1)$-invariant domain Ω in $\mathbb{R}^N$, $1 \leq k \leq N-3$, at the $(k+1)$-st critical exponent $p = \frac{2(N-k)}{N-k-2}$, for any $\lambda \in \mathbb{R}$. We show that $O(k+1)$-invariant ground states exist for λ in some interval to the left hand side of each $O(k+1)$-eigenvalue, and that no $O(k+1)$-invariant ground states exist in some interval $(-\infty, \lambda_*)$ with $\lambda_* > 0$ if $k \geq 2$.

This question is related to the existence of ground states to the anisotropic critical problem

$$-\mathrm{div}(a(x)\nabla u) = \lambda b(x) u + c(x)\, |u|^{2^*-2}\, u \quad \text{in } \Theta, \qquad u = 0 \quad \text{on } \partial\Theta,$$

where a, b, c are positive continuous functions on $\overline{\Theta}$. We give a minimax characterization for the ground states of this problem, study the ground state energy level as a function of λ, and obtain a bifurcation result for ground states.

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