Semi-classical bound states for NLS : concentration on circles

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In Quantum Mechanics, the nonlinear Schrödinger equation (NLS) with a magnetic field B, having source in A, and a scalar (electric) potential U has the form

$$i\hbar \frac{\partial \psi}{\partial t} = (i\hbar\nabla + A(x))^2 \psi + U(x)\psi = f(|\psi|^2)\psi, \quad x \in \mathbb{R}^3$$

Many efforts in the past years have been devoted to the study of semi-classical standing waves, namely solutions of the form $\psi(x,t) = e^{-\frac{E}{\hbar}t}\, u(x)$ assuming $\hbar$ is small. When $A \equiv 0$, the existence of solutions concentrating around some point (or a set of points) or more generally around a manifold, has been proved by elaborated methods.

When $A \not\equiv 0$, it has been shown by Kurata, Cingolani and Cingolani-Secchi that the magnetic field does not play any role for solutions that concentrate around a single point or around multiple points. Namely, the points where the concentration occurs are not determined by the magnetic potential and their location only depends on the critical points of the electric potential.

In this talk, I will discuss a new class of solutions in the case $A \not\equiv 0$. Namely, solutions that concentrate on circles. The new feature is that the position of the circle depends on both the electric and the magnetic potentials.

The talk is based on a joint work with J. Di Cosmo & J. Van Schaftingen and a joint work with S. Cingolani & M. Nys.