Large conformal metrics with prescribed sign-changing Gauss curvature

Carlos Román

Université Pierre et Marie Curie
I@ann.jussieu.fr

Let (M, g) be a two dimensional compact Riemannian manifold of genus $g(M) > 1$. Let f be a smooth function on M such that

$$f \geq 0, \quad f \not\equiv 0, \quad \min_M f = 0.$$

Let $p_1, ..., p_n$ be any set of points at which $f(p_i) = 0$ and $D^2 f(p_i)$ is non-singular. We prove that for all sufficiently small $\lambda > 0$ there exists a family of "bubbling" conformal metrics $g_\lambda = e^{u_\lambda} g$ such that their Gauss curvature is given by the sign-changing function $K_{g_\lambda} = -f + \lambda^2$. Moreover, the family u_λ satisfies

$$u_\lambda(p_j) = -4 \log \lambda - 2 \log \left(\frac{1}{\sqrt{2}} \log \frac{1}{\lambda} \right) + O(1)$$

and

$$\lambda^2 e^{u_\lambda} \rightharpoonup 8\pi \sum_{i=1}^{n} \delta_{p_i}, \quad \text{as } \lambda \to 0,$$

where δ_p designates Dirac mass at the point p. This is joint work with Manuel del Pino.