On dispersion generalized Benjamin-Ono equations

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Abstract

We consider the Cauchy problem related to the family of k-dispersion generalized Benjamin-Ono (k-DGBO) equations:

$$\begin{cases} u_t + D_x^\alpha u_x + \mu\, u^k u_x = 0, \quad (t,x) \in \mathbb{R}\times\mathbb{R}, \\ u(0,x) = u_0(x), \end{cases} \tag{1}$$

where $u = u(t,x)$ is real-valued, $\alpha \in [1,2]$, $\mu \in \{\pm 1\}$ and $k \in \mathbb{Z}^+$. Here, D_x^α represents the 1-dimensional fractional Laplacian operator in the spatial variable x. For $k \geq 4$, we establish local and global well-posedness results for (1) in both the critical $\left(s = \frac{k-2\alpha}{2k}\right)$ and subcritical $\left(s > \frac{k-2\alpha}{2k}\right)$ regimes, addressing sharp regularity in homogeneous and inhomogeneous Sobolev spaces. Additionally, our method enables the formulation of a scattering criterion and a scattering theory for small data. We also investigate the case $k = 3$ via frequency-restricted estimates, obtaining local well-posedness results for the initial value problem associated with the 3-DGBO equation and generalizing the existing results in the literature for the whole subcritical range. For higher dispersion, these local results can be extended globally even for rough data, particularly for initial data in Sobolev spaces with negative indices. As a byproduct, we derive new nonlinear smoothing estimates. This is a joint work with Luccas Campos (UFMG) and Felipe Linares (IMPA).