Classifying horocycle ergodic measures on translation surfaces

Samantha Fairchild
Question posed by Jon Chaika

University of Washington   skayf@uw.edu

University of Utah   chaika@math.utah.edu

Translation surface in $\mathcal{H}(2)$

- Opposite sides identified by translation
  (Riemann surface with one-form dz)

Translation surface in $\mathcal{H}(2)$

- One cone point (zero of the one-form).
  Total angle is $6\pi = 2\pi + 2(2\pi)$ so 2 is order of singularity.

Translation surface in $\mathcal{H}(2)$
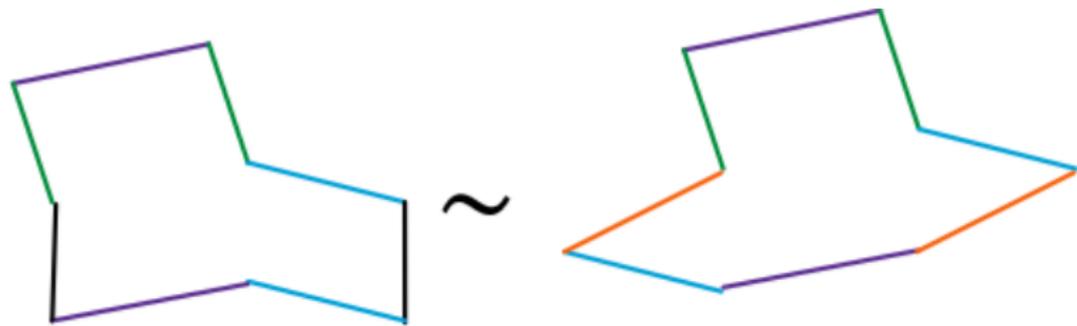

- Defined up to cut-and-paste equivalence.

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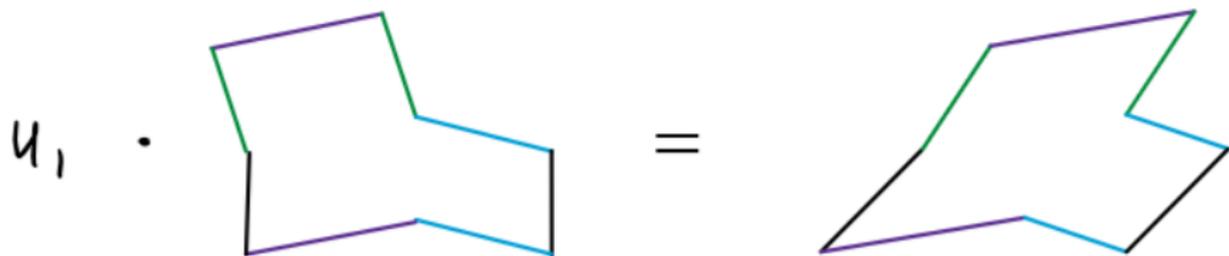- Defined up to cut-and-paste equivalence.

Translation surface in $\mathcal{H}(2)$

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$SL(2, \mathbb{R})$ action on the translation surface linearly on $\mathbb{R}^2$

$$u_t = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \qquad u_t \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a + tb \\ b \end{pmatrix}$$

The Question

Does every u_t-ergodic measure in $\mathcal{H}(2)$ that gives weight zero to translation surfaces with a horizontal saddle connection $SL(2, \mathbb{R})$-ergodic?

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- Horizontal saddle connections preserved by $u_t \implies$ supported on strict subset of $SL(2, \mathbb{R})$-orbit

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- Exotic horocycle ergodic measures are not known in $\mathcal{H}(2)$.
  - Are there two different measures with the same support?
  - Are there horocycle ergodic measures with fractional Hausdorff dimension?

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- For other strata, the answer is "no"
  - There are examples of horocycle-ergodic measures which are not $SL(2,\mathbb{R})$-invariant.