CONCENTRATION ON SUBMANIFOLDS FOR AN AMBROSETTI-PRODI TYPE PROBLEM

FETHI MAHMOUDI*

Abstract

Given a smooth bounded domain Ω of $\mathbb{R}^n$ and consider the problem

$$\begin{cases} -\Delta u = |u|^p - t\,\psi & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{0.1}$$

where t is a large positive parameter, $p > 1$ and ψ is an eigenfunction of $-\Delta$ with Dirichlet boundary condition corresponding to the first eigenvalue λ_1. Assuming that Ω contains a k-dimensional compact submanifold K which is stationary and non-degenerate for the weighted functional

$$\int_K \psi^{(1-\frac{1}{p})(\frac{p+1}{p-1}-\frac{n-k}{2})} dvol$$

such that $\mathrm{dist}(K, \partial\Omega) > \delta_0 > 0$ then for $1 < p < \frac{n+2-k}{n-2-k}$ we prove the existence of a sequence $t = t_j \to \infty$ and solutions u_t that concentrate along K. This result proves in particular the validity of a conjecture by Hollman-Mckenna in full generality.

Work in Collaboration with Z. Khemiri and A. Messaoudi