SECOND WORKSHOP ON EVOLUTION EQUATIONS IN VALDIVIA 2016 AT U. AUSTRAL

DETAILED ABSTRACTS

PROGRAM II WORKSHOP EVOLUTION EQUATIONS VALDIVIA 2016

	Monday 12	Tuesday 13	Wednesday 14	Thrusday 15	Friday 16
8:30-9:00	Registration Welcome				
9:00-9:50	Jean-Claude Saut	Frank Merle	Luis Vega	Pierre Raphaël	Herbert Koch
10:00-10:30	Coffe Break	Coffe Break	Coffe Break	Coffe Break	Coffe Break
10:30-11:20	Hajer Bahouri	Oana Ivanovici	Mathieu Lewin	Piotr Bizon	Kenji Nakanishi
11:30-12:20	Zaher Hani	Fabrice Planchon	Andrew Lawrie	Felipe Linares	Frederic Rousset
12:30-14:00	Lunch	Lunch	Touristic Visit	Lunch	Closing words
14:00-14:50	Anne-Sophie de Suzzoni	Marjolaine Puel		Juan Dávila	
15:00-15:30	Coffe Break	Coffe Break		Coffe Break	
15:30-16:20	Miguel Alejo	Julien Sabin		Fethi Mahmoudi	
16:30-17:20	Didier Pilod	Nejla Nouaili Jacek Jendrej		Charles Collot Yang Lan	

Abstracts

Miguel A. Alejo

On the Variational Structure of Breather solutions

In this talk I will show some recent results about the variational structure and stability properties for breathers solutions in different nonlinear models and I will support them with numerical results, computing the discrete spectra of the linearized operators around breather solutions of some nonlinear PDEs.

Hajer Bahouri

Dispersive estimates for the Schrdinger equation on 2-step stratified Lie groups

The present work is dedicated to the proof of dispersive estimates on 2-step stratified Lie groups, for the linear Schrödinger equation involving a sublaplacian. It turns out that the Schrödinger propagator on 2-step stratified Lie groups behaves like a wave operator on a space of the same dimension as the center of the group and like a Schrödinger operator on a space of the same dimension as the radical of the canonical skew-symmetric form. This unusual behavior of the Schrödinger propagator makes the analysis of the explicit representation of the solutions tricky and gives rise to uncommon dispersive estimates. It will also appear from our analysis that the optimal rate of decay is not always in accordance with the dimension of the center as it is the case for H-type groups: we will exhibit examples of 2-step stratified Lie groups with center of any dimension and for which no dispersion phenomenon occurs for the Schrödinger equation. We will identify a generic condition under which the optimal rate of decay is achieved.

Piotr Bizon

Conformal flow on the 3-sphere

For the conformally invariant cubic wave equation on the 3-sphere we construct an effective infinite-dimensional time-averaged dynamical system that approximates the dynamics of small solutions on long timescales. This effective system, which we call the conformal flow, was shown to display a rich phenomenology, including low-dimensional invariant subspaces, a wealth of stationary states, and periodic energy flows with alternating direct and inverse cascades. I will describe these results, as well as close parallels between the conformal flow and the cubic Szëgo equation.

Charles Collot

On the stability of non-ODE blow-up for the energy supercritical semilinear heat equation

Two mechanisms are responsible for singularity formation at the origin for solutions to the focusing semilinear heat equation with power nonlinearity in the radial case. The first one is the concentration in finite time of stationary states by scale instability (type II blow-up), and the second one is the concentration in finite time of backward self-similar solutions. The latter involve a profile which shrinks according to the scaling law of the equation and at the diffusion speed. An example is given by solutions which are constant in space and which tend to infinity in finite as they solve the corresponding nonlinear ODE. In a range of parameters for which the equation is in the so called energy supercritical regime, Budd and Qi, Budd and Norbury, Troy, Lepin and Mizoguchi investigated the existence of backward self-similar solutions which are not constant in space. In some cases there exists a countable family of such radial solutions. In a joint work with Raphaël and Szeftel we gave an alternative proof for the existence of these solutions which gave us tools to show the conditional non-radial and nonlinear stability of the underlying blow-up phenomenon.

Juan Dávila

Finite time blowup for the harmonic map flow in 2 dimensions

We study singularity formation in the harmonic map flow from a two dimensional domain into the sphere. We show that for suitable initial conditions the flow develops a type II singularity at some point in finite time, and obtain the rate and profile. We show also that this is stable under small perturbations of the initial condition. The the rate and profile of blow up was derived formally by van den Berg, Hulshof and King (2003) and proved by Raphael and Schweyer (2013) in the class of 1-corrotationally symmetric maps. This is joint work with Manuel del Pino (Universidad de Chile) and Juncheng Wei (University of British Columbia).

Anne Sophie de Suzzoni

The relativistic dynamics of an electron coupled with a classical nucleus. Joint work with F. Cacciafesta, D. Noja and E. Séré

This talk is about the Dirac equation. We consider an electron modeled by a wave function and evolving in the Coulomb field generated by a nucleus. In a very rough way, this should be an equation of the form

$$i\partial_t u = -\Delta u + V(\cdot q(t))u$$

where u represents the electron while $q(t)$ is the position of the nucleus. When one considers relativistic corrections on the dynamics of an electron, one should replace the Laplacian in the equation by the Dirac operator. Because of limiting processes in the chemistry model from which this is derived, there is also a cubic term in u as a correction in the equation. What is more, the position of the nucleus is also influenced by the dynamics of the electron. Therefore, this equation should be coupled with an equation on q depending on u.

I will present this model and give the first properties of the equation. Then, I will explain why it is well-posed on H^2 with a time of existence depending only on the H^1 norm of the initial datum for u and on the initial datum for q. The linear analysis, namely the properties of the propagator of the equation $i\partial_t u = Du + V(\cdot q(t))$ where D is the Dirac operator is based on works by Kato, while the non linear analysis is based on a work by Cancès and Lebris.

It is possible to have more than one nucleus. I will explain why.

Zaher Hani

TBA

Oana Ivanovici

TBA

Jacek Jendrej

On two-bubble solutions for energy-critical dispersive equations

The Soliton Resolution Conjecture predicts that, generically, solutions of nonlinear dispersive equations decompose asymptotically into a superposition of a finite number n of solitons and a linear radiation term. In the case of absence of the radiation term, such a solution is called a pure multi-soliton or a pure n-soliton. Motivated by the recent progress on this conjecture for energy-critical equations, I consider the problem of existence of pure radial two-solitons for the energy critical wave equation and the energy-critical Schrödinger equation with a focusing power nonlinearity.

Herbert Koch

Conserved energies for NLS, mKdV and KdV.

In the talk I will explain the construction of a family of conserved energies for all three equations and consequences.

Yang Lan

Stable self-similar blow up dynamics for slightly L^2 supercritical gKdV equations

We consider the focusing generalized KdV equations with slightly L^2 supercritical nonlinearity. We will use the self-similar profile constructed by H. Koch to prove the existence and stability of a blow up dynamics with self-similar blow up rate in the energy space H^1. We will also give a specific description of the formation of singularity near the blow up time.

Andrew Lawrie

Energy subcritical nonlinear wave equations.

In this talk well describe recent joint work with B. Dodson on the energy subcritical radial cubic wave equation and forthcoming work with Dodson, Mendelson, and Murphy on the same equation in the non-radial setting. We prove that all solutions scatter as long as the critical norm of the evolution stays bounded using technique inspired by the work of Kenig and Merle and Duyckaerts, Kenig, and Merle. Well focus on the new methods we introduced to treat the energy subcritical case and on how our results complement the classic work of Merle and Zaag in this setting.

Mathieu Lewin

Long time dynamics for the Hartree equation

This talk will be a review of known results and open problems concerning the nonlinear Hartree equation used to describe the electrons in an atom or a molecule. It is a dispersive equation of Schrödinger type with (long range) Coulomb forces in 3D. We will discuss the existence of stationary states, formulate a "soliton resolutio" type conjecture and present a theorem proved with Enno Lenzmann on the long time dynamics, based on novel Virial type arguments. If time permits, we will also mention open problems for the infinite Coulomb plasma.

Felipe Linares

On the fractional KP equation.

We will discuss a recent result regarding local well-posedness for the fractional KP equation. This is a joint work with D. Pilod (UFRJ, Brazil) and J-C. Saut (Orsay).

Fethi Mahmoudi

Concentration on submanifolds for an Ambrosetti-Prodi type problem

Abstract: see `http://tinyurl.com/gppc76c`

Frank Merle

An example of insolatedness of caracteristic points for the nonlinear wave equation in dimension two

We consider the semilinear wave equation with subconformal power nonlinearity in two space dimensions. We construct a finite-time blow-up solution with an isolated characteristic blow-up point at the origin, and a blow-up surface which is centered at the origin and has the shape of a stylized pyramid, whose edges follow the bisectrices of the axes in R^2. The blow-up surface is differentiable outside the bisectrices. As for the asymptotic behavior in similarity variables, the solution converges to the classical one-dimensional soliton outside the bisectrices. On the bisectrices outside the origin, it converges (up to a subsequence) to a genuinely two-dimensional stationary solution, whose existence is a by-product of the proof. At the origin, it behaves like the sum of 4 solitons localized on the two axes, with opposite signs for neighbors. This is the first example of a blow-up solution with a characteristic point in higher dimensions, showing a really two-dimensional behavior. Moreover, the points of the bisectrices outside the origin give us the first example of non-characteristic points where the blow-up surface is non-differentiable.

Kenji Nakanishi

Scattering for the Gross-Pitaevskii equation in the 3D radial energy space

This is joint work with Zihua Guo and Zaher Hani. We consider long-time behavior of solutions for the Gross-Pitaevskii equation (GP), or the nonlinear Schrödinger equation (NLS) with non-zero constant amplitude at spatial infinity, in three space dimensions. The main result is the scattering for small initial data in the energy space with radial symmetry or angular regularity. For NLS, it means asymptotic stability for small energy perturbation (with the symmetry) of the plane wave solutions. The interaction with the plane wave is very long range, which makes the scattering for GP much harder than NLS. We introduce a quadratic transform to remove its effect around zero frequency, which is slightly different from those in the previous work of Gustafson, Tsai and myself. After the transform, we can make a global iteration by the Strichartz estimate for the linearized equation which is improved under the symmetry. The scattering can not extend to the entire energy space, since GP admits traveling wave solutions. Under the radial symmetry, however, one might expect large-data scattering, as the traveling waves are not radial. Concerning this question, we have an interesting observation that the focusing energy-critical wave equation appears in the zero-frequency limit, which suggests that its ground state might be the lowest energy obstruction for the scattering of GP.

Nejla Nouali

Construction of blow-up solution for complex Ginzburg-Landau equation in the critical case

We construct a solution for the complex Ginzburg-Landau equation in the critical case, which blows up in finite time T only at one blow-up point. We also give a sharp description of its profile. The proof relies on the reduction of the problem to a finite dimensional one, and the use of index theory to conclude.

Didier Pilod

Construction of a minimal mass blow up solution of the modified Benjamin-Ono equation

This talk is based on a joint work with Yvan Martel (Ecole Polytechnique)

We construct a minimal mass blow up solution of the modified Benjamin-Ono equation (mBO), which is a classical one dimensional nonlinear dispersive model.

Let $Q \in H^{\frac{1}{2}}$, $Q > 0$, be the unique ground state solution associated to mBO. We show the existence of a solution S of mBO satisfying $\|S\|_{L^2} = \|Q\|_{L^2}$ and

$$S(t) - \frac{1}{\lambda^{\frac{1}{2}}(t)} Q\left(\frac{\cdot - x(t)}{\lambda(t)}\right) \to 0 \quad \text{in } \ H^{\frac{1}{2}}(\mathbb{R}) \text{ as } \ t \downarrow 0,$$

where

$$\lambda(t) \sim t, \quad x(t) \sim -|\ln t| \quad \text{and} \quad \|S(t)\|_{\dot{H}^{\frac{1}{2}}} \sim t^{-\frac{1}{2}} \|Q\|_{\dot{H}^{\frac{1}{2}}} \quad \text{as } t \downarrow 0.$$

This existence result is analogous to the one obtained by Martel, Merle and Raphaël (J. Eur. Math. Soc., 17 (2015)) for the mass critical generalized Korteweg-de Vries equation (gKdV). However, in contrast with the gKdV equation, for which the blow up problem is now well-understood in a neighborhood of the ground state, S is the first example of blow up solution for mBO.

The proof involves the construction of a blow up profile, energy estimates as well as refined localization arguments, developed in the context of Benjamin-Ono type equations by Kenig, Martel and Robbiano (Ann. Inst. H. Poincaré, Anal. Non Lin., 28 (2011)). Due to the lack of information on the mBO flow around the ground state, the energy estimates have to be considerably sharpened here.

Fabrice Planchon

TBA

Marjolaine Puel

Asymptotic limits for collisional kinetic equations

In several domain of applied math as nuclear industry, aerodynamic, biology, gas dynamics may be modeled by some kinetic equations. Their structure is complex and a real challenge consists in providing simpler models that are more performant for numerics. We first try to explain how kinetic equations may be linked to particle trajectories and introduce two particular cases, the Boltzmann equation and the Fokker Planck equation. Then we will give the context in which kinetic equations may be approximated by more macroscopic equations. At the end, we will focus on the diffusion approximation and in particular on the anomalous diffusion approximation for both Boltzmann and Fokker Planck.

Pierre Raphaël

TBA

Frédéric Rousset

Quasineutral limit for the Vlasov-Poisson system

We will study the Vlasov Poisson system for electrons or ions in the quasineutral regime. In this regime, there is a small parameter in front of the Laplacian in the Poisson equation and the aim is to describe the limit when this parameter tends to zero. This is a singular limit where many instabilities occur, in particular the limit system is not always well-posed. We will describe recent results obtained with D. Han-Kwan about the justification of this limit under some stability conditions.

Julien Sabin

Maximizers for the Stein-Tomas inequality

We give a necessary and sufficient condition for the precompactness of all optimizing sequences for the Stein–Tomas inequality. In particular, if a well-known conjecture about the optimal constant in the Strichartz inequality is true, we obtain the existence of an optimizer in the Stein–Tomas inequality. Our result is valid in any dimension. This is a joint work with Rupert Frank (Caltech) and Elliott Lieb (Princeton).

Jean-Claude Saut

Long time existence for some water wave models

Most of dispersive equations or systems are not derived from first principles but as asymptotic models derived to zoom at some specific regimes of amplitudes, wavelengthsin order to explain the dynamics of more complex systems. They are not supposed to be "good" models for all time but only on long time scales, in term of inverse powers of a small parameter. Those long time issues cannot be solved in general by using the elaborate "dispersive" techniques that have been developed to study the local Cauchy problem in large spaces and one should use instead kind of "hyperbolic" techniques. This will be illustrated on various dispersive systems arising in the theory of water waves.

Luis Vega

TBA